SOME SUPPLEMENTS TO THE GUYON-MASSONNET METHOD OF COMPUTING BEAM GRILLAGES

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1. INTRODUCTION

One of the most suitable methods of computing bridge-type beam grillages, which is based on the analogy with an orthotropic plate, is the one worked out by Guyon and Massonnet. This method uses the so-called distribution coefficients introduced by Guyon [3] in the computation of isotropic plates. The procedure is rapid, well-arranged, easy to check and curves of coefficients for the computation of various static values can be simply plotted or tabulated.

In accordance with relations derived by Massonnet [6] a graphical representation was given by Little [5] of the coefficients K for the computation of deflections and longitudinal bending moments and of the coefficients μ for the computation of cross bending moments, provided the Poisson factor is zero.

Rowe [7] plotted these coefficients for the case when the Poisson factor $\eta = 0{\cdot}15$. In both cases the representation was made for a cross bracing parameter ranging from zero to two.

The author calculated the values of all of these coefficients [1] also for the cross bracing parameter ϑ ranging from 2·0 to 5·0 so that the computations of static values may be improved by considering more members of the corresponding series.[1])

When computating grillages it is usually necessary to know the magnitude and variation of shear forces in both directions and reactions. Therefore, the author derived relations for determining these quantities, computed numerical values of the corresponding coefficients and plotted them as charts in dependence on the magnitude of cross bracing parameter ϑ, loading excentricity e and position y of the cross section in which the effect is searched for.

[1]) As a matter of fact, it was proved that the m-th coefficient of the series, which corresponds to the load distribution $p_m = p_{0m} \sin (m\pi x/l)$, is equal to the first member of a series for a structure with m-times more pliable cross bracing, i.e. with a flexural rigidity parameter $m\vartheta$. In other words, for a loading $p_m = p_{0m} \sin (m\pi x/l)$ the cross bracing becomes m-times more pliable than for $p = p_1 \sin (\pi x/l)$.

The determination of shear forces requires, with regard to the effect of twisting moments, to distinguish between a grillage and an actual orthotropic plate (realized for example by a slab bridge or a bridge built of prefabricated cross-prestressed units).

2.1 Longitudinal shear forces in a plate

The shear force in direction X per unit width, using the notation in fig. 1, is given by the relation

$$(1)\quad Q_x = -\varrho_T \frac{\partial^3 w}{\partial x^3} - 2\gamma \frac{\partial^3 w}{\partial x\, \partial y^2} = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = \frac{\partial}{\partial x}\left(M_x + \frac{2\gamma}{\varrho_P} M_y\right).$$

Introducing $2\gamma = \alpha \sqrt{(\varrho_T \varrho_P)}$ and rearranging, we obtain

$$(2)\quad Q_x = \sum_{m=1,2,3\ldots} p_{0m}\varepsilon_{\alpha m} \frac{l}{b} \cos\frac{m\pi x}{l},$$

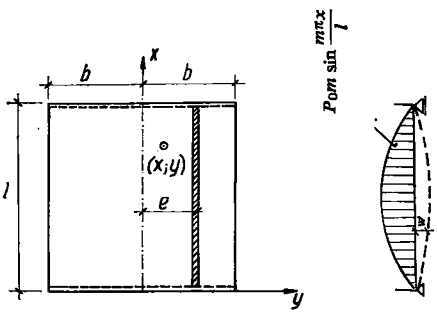

Fig. 1.

where

$$(3)\quad \varepsilon_{\alpha m} = \frac{m^3 \varrho_T \pi^3 b}{l^4 p_{0m}} \left[(1-\alpha^2)\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \right.$$

$$+ \alpha\sqrt{(1-\alpha^2)}\left(A_m N_{\varphi m} - \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} M_{\varphi m}\right) +$$

$$+ (1-\alpha^2)\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) +$$

$$+ \alpha\sqrt{(1-\alpha^2)}\left(-C_m P_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} O_{\varphi m}\right) +$$

$$\left. + \bar{C}_m(1+\alpha)\left(O_{|\varphi-\psi|m} + \sqrt{\left(\frac{1-\alpha}{1+\alpha}\right)} P_{|\varphi-\psi|m}\right)\right],$$

in which denotes:

(4)
$$M_{\varphi m} = e^{m\varphi n'} \cos m\varphi t' ,$$
$$N_{\varphi m} = e^{m\varphi n'} \sin m\varphi t' ,$$
$$O_{\varphi m} = e^{-m\varphi n'} \cos m\varphi t' ,$$
$$P_{\varphi m} = e^{-m\varphi n'} \sin m\varphi t' ,$$
$$O_{|\varphi-\psi|m} = e^{-m|\varphi-\psi|n'} \cos m|\varphi - \psi|\, t' ,$$
$$P_{|\varphi-\psi|m} = e^{-m|\varphi-\psi|n'} \sin m|\varphi - \psi|\, t' ,$$
$$n' = \vartheta \sqrt{\left(\frac{1+\alpha}{2}\right)}; \quad t' = \vartheta \sqrt{\left(\frac{1-\alpha}{2}\right)},$$
$$\varphi = \frac{\pi y}{b}; \quad \psi = \frac{\pi e}{b},$$
$$\bar{C}_m = \frac{p_{0m} b^3}{2\, \varrho_P \pi^3 \vartheta^3 m^3 \sqrt{[2(1+\alpha)]}},$$

A_m, B_m, C_m, D_m are integration constants which are determined from the boundary conditions [6],

cross bracing parameter

$$\vartheta = \frac{b}{l} \sqrt[4]{\left(\frac{\varrho_T}{\varrho_P}\right)},$$

twisting parameter

$$\alpha = \frac{\gamma_T + \gamma_P}{2\sqrt{(\varrho_T + \varrho_P)}},$$

b half width of structure, l span of structure, ϱ_T, γ_T, ϱ_P, γ_P (bending) twisting rigidity in direction of beams (of cross beams) per unit length (fig. 1), p_m amplitude of m-th member of the Fourier series for actual loading.

For a slab unrigid in torsion, e.g. for $\alpha = 0$, a serviceable analogy may be used, which supposes that a cross beam of differential width can be considered as a beam on elastic foundation [4]. The force Q_x can in this case be again expressed by the equation

(2a)
$$Q_x = \sum_{m=1,3,5\ldots} p_{0m} \varepsilon_{0m} \frac{l}{b} \cos \frac{m\pi x}{l},$$

where ε_{01} is given by

(5)
$$\varepsilon_{01} = \frac{\vartheta}{\sqrt{2}} \frac{1}{(\mathrm{sh}^2\, 2\lambda b - \sin^2 2\lambda b)} \{[2\, \mathrm{ch}\, \lambda(b \pm y) \cos \lambda(b \pm y)]\, S_e +$$
$$+ [\mathrm{ch}\, \lambda(b \pm y) \sin \lambda(b \pm y) + \mathrm{sh}\, \lambda(b \pm y) \cos \lambda(b \pm y)]\, T_e\} .$$

In this equation

(6) $$S_e = \left[\operatorname{sh} 2\lambda b \cos \lambda(b \pm e) \operatorname{ch} \lambda(b \mp e) - \sin 2\lambda b \operatorname{ch} \lambda \times \times (b \pm e) \cos \lambda(b \mp e)\right],$$

$$T_e = \{\operatorname{sh} 2\lambda b[\sin \lambda(b \pm e) \operatorname{ch} \lambda(b \mp e) - \cos \lambda(b \pm e) \operatorname{sh} \lambda(b \mp e)] + + \sin 2\lambda b[\operatorname{sh} \lambda(b \pm e) \cos \lambda(b \mp e) - \operatorname{ch} \lambda(b \pm e) \sin \lambda(b \mp e)]\}$$

and $\lambda = (\pi\vartheta)/(b\sqrt{2})$.

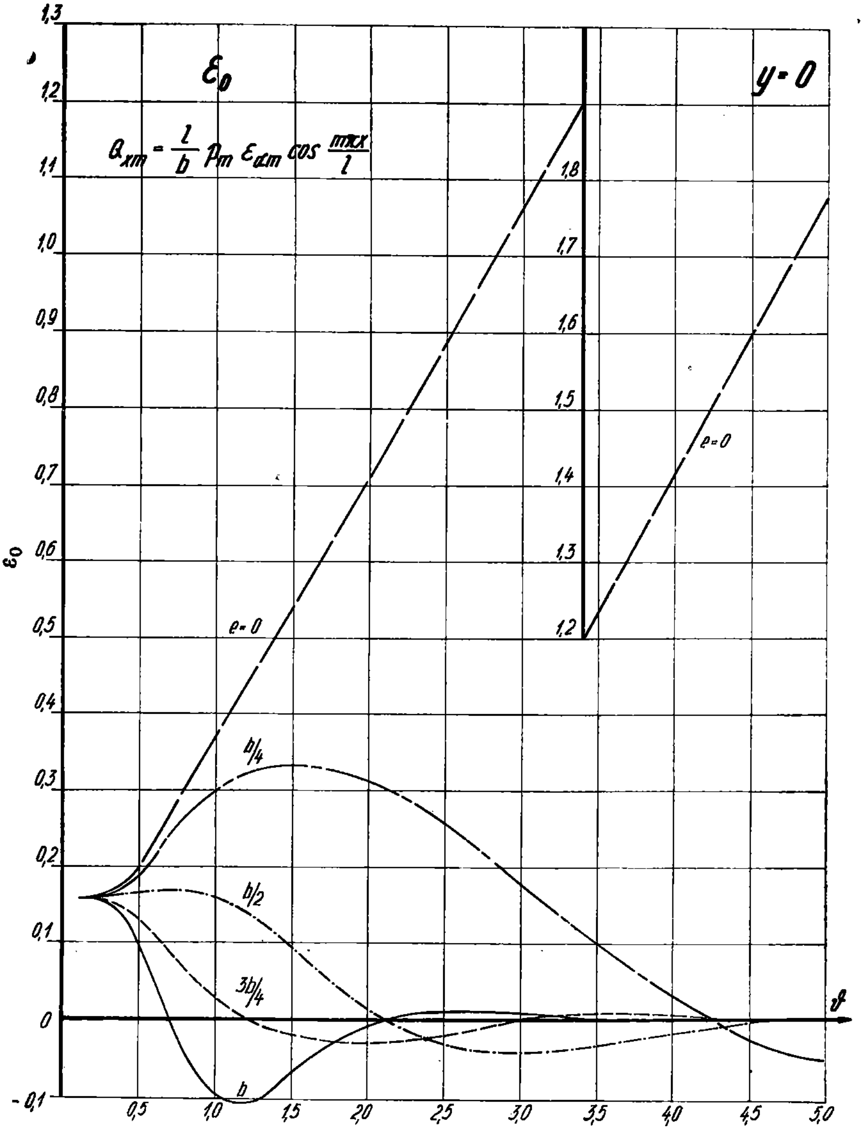

Fig. 2.

The upper signs in parentheses hold for $e \geqq y$, the lower ones for $e \leqq y$. The coefficient ε_{0m} is obtained from eq. (5) by substitution of $m\vartheta$ for ϑ and $m\lambda$ for λ.

In calculating the coefficient ε_1 for the case when $\alpha = 1$ we can start out from the relations derived by Guyon [3] for an isotropic plate. Expressing again the shear force in the form

(2b) $$Q_x = \sum_{m=1,2,3\ldots} p_{0m}\varepsilon_{1m} \frac{l}{b} \cos \frac{m\pi x}{l};$$

the value of ε_{11} is

(8) $$\varepsilon_{11} = \frac{\vartheta}{4\,\mathrm{sh}^2\,\sigma}\left[2\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\chi + (2\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\varphi)\,U_\psi + (2\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\varphi)\,V_\psi\right],$$

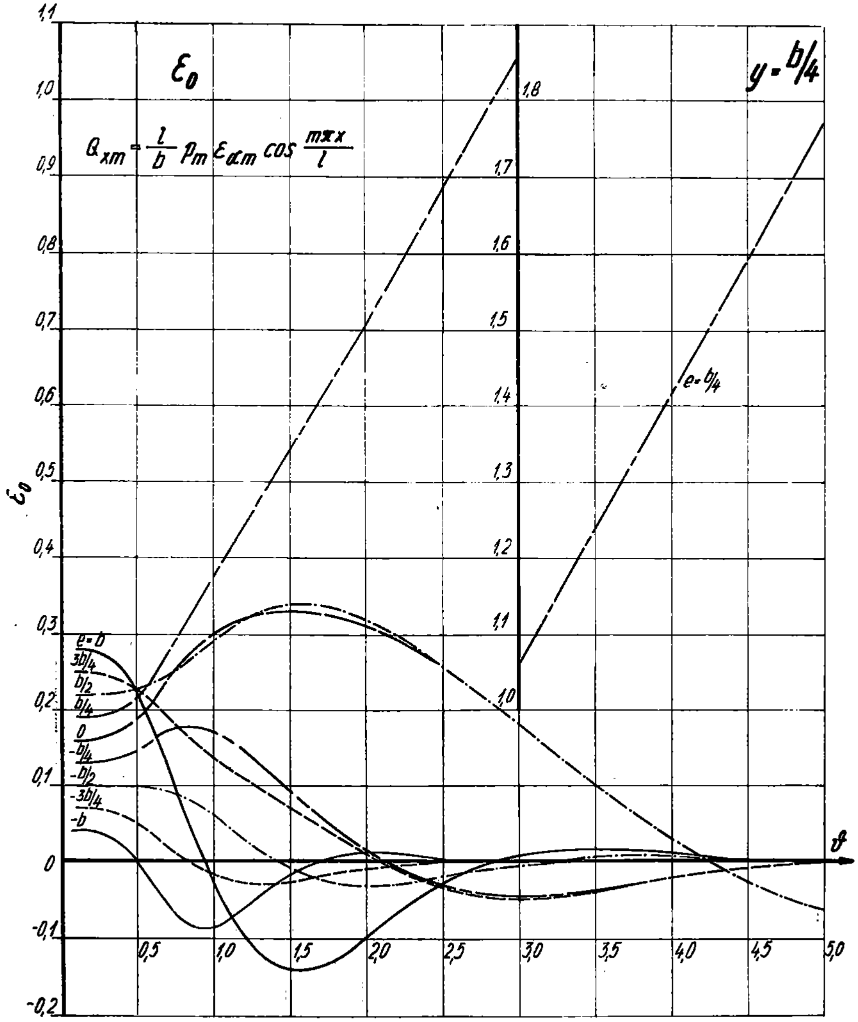

Fig. 3.

where

$$U_\psi = \frac{[(\sigma \operatorname{ch} \sigma - \operatorname{sh} \sigma) \operatorname{ch} \vartheta\psi - \vartheta\psi \operatorname{sh} \sigma \operatorname{sh} \vartheta\psi]}{3 \operatorname{sh} \sigma \operatorname{ch} \sigma - \sigma}, \tag{9}$$

$$V_\psi = \frac{[(2 \operatorname{sh} \sigma + \sigma \operatorname{ch} \sigma) \operatorname{sh} \vartheta\psi - \vartheta\psi \operatorname{sh} \sigma \operatorname{ch} \vartheta\psi]}{3 \operatorname{sh} \sigma \operatorname{ch} \sigma + \sigma},$$

$\sigma = \vartheta\pi$ and $\chi = \pi - |\varphi - \psi|$.

ε_{1m} can be obtained from eq. (8) by substitution of $m\vartheta$ for ϑ and $m\sigma$ for σ.

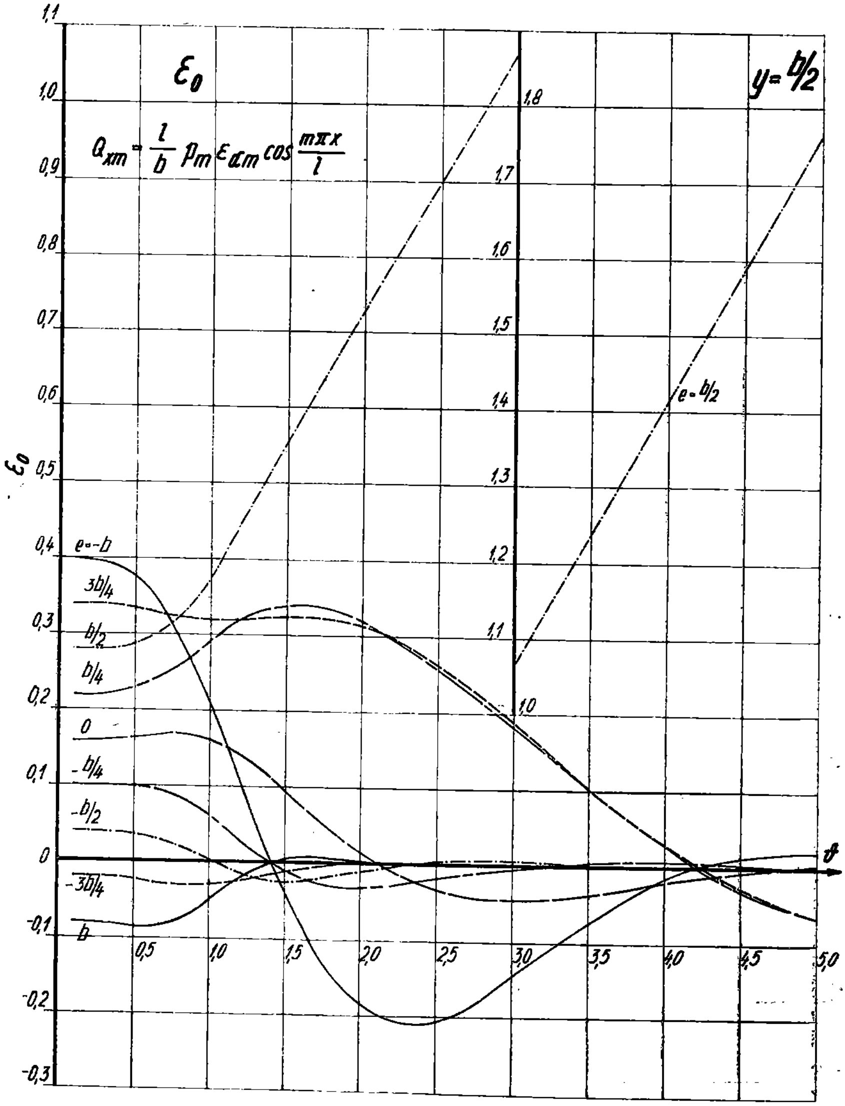

Fig. 4.

The values of coefficients ε_α for various values of α and $\vartheta = 0{,}66874$ were calculated by using eq. (3), see [2]. By analysis of the variation of these coefficients in dependence on α were derived two interpolation formulae (holding for discrete values) for

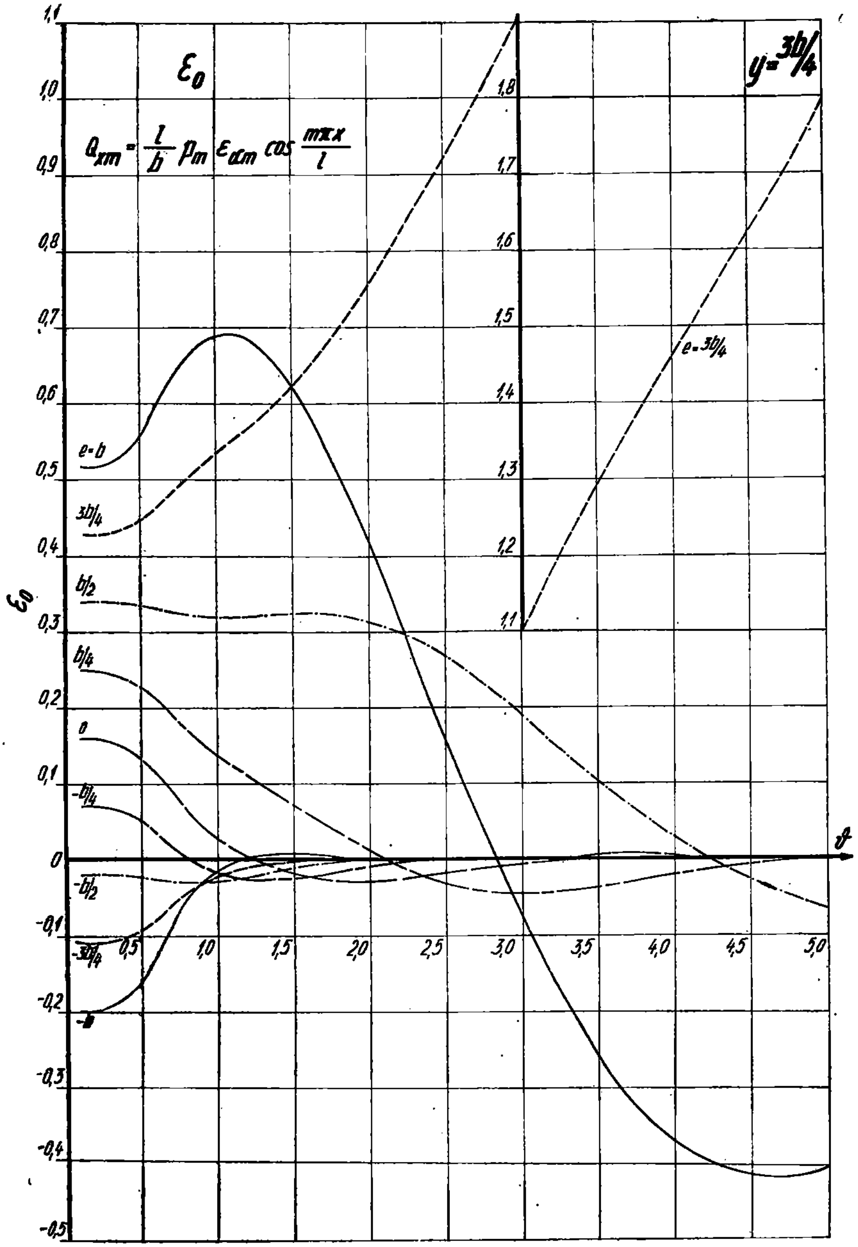

Fig. 5.

the determination of a general value of $\varepsilon_{\alpha m}$ by means of ε_{0m} and ε_{1m} in dependence on the position of loads and cross section in which the effect is searched for.

It holds that

$$(10)\qquad \begin{cases} \text{for } |y| + |e| \leqq \frac{3}{4}b \text{ is } \varepsilon_{\alpha m} = \varepsilon_{0m} + (\varepsilon_{1m} - \varepsilon_{0m})\,\alpha \\ \text{and for } |y| + |e| > \frac{3}{4}b \text{ is } \varepsilon_{\alpha m} = \varepsilon_{0m} + (\varepsilon_{1m} - \varepsilon_{0m})\sqrt{\alpha}\,. \end{cases}$$

Numerical values of the coefficients ε_{0m} and ε_{1m} follow from the diagrame in fig. 2 to 11.

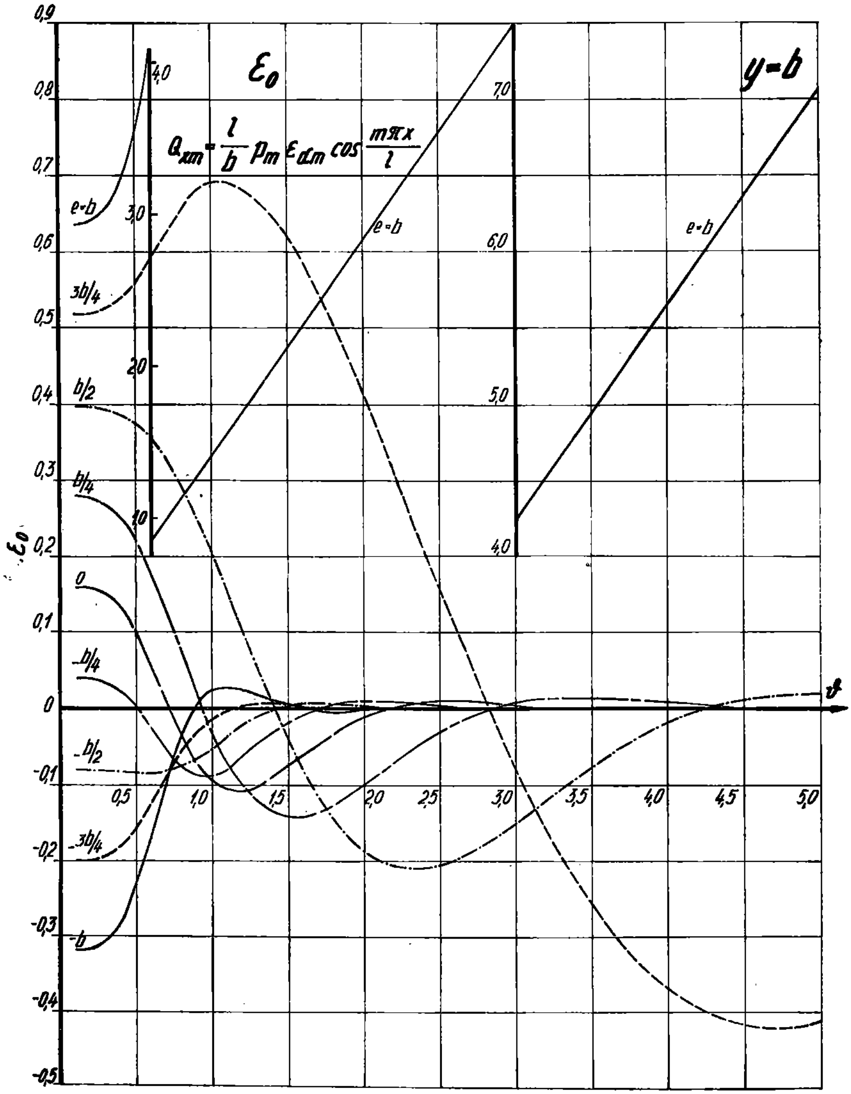

Fig. 6.

2.2 Shear forces in longitudinal beams in a grillage

The shear force in direction X per unit width is given by the expression

(11) $$Q_x = -\varrho_T \frac{\partial^3 w}{\partial x^3} - \gamma_P \frac{\partial^3 w}{\partial x\, \partial y^2} = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = \frac{\partial}{\partial_x}\left(M_x + \frac{\gamma_P}{\varrho_P} M_y\right).$$

By substitution for M_x and M_y and rearrangement we obtain

(12) $$Q_x = \sum_{m=1,2,3\ldots} p_{0m} \cos \frac{m\pi x}{l}\left(\frac{l}{2b\pi m} K_{\alpha m} + \frac{\gamma_P}{\varrho_P}\frac{m\pi b}{l}\mu_{\alpha m}\right),$$

where $K_{\alpha m}$ is the coefficient used in calculating deflections or longitudinal bending moments, $\mu_{\alpha m}$ the coefficient used for calculation of cross bending moments.

These coefficients have the form:

(13) $$K_{\alpha m} = \frac{2b\varrho_T\pi^4 m^4}{p_{0m} l^4}\left[\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) + \bar{C}_m\left(O_{|\varphi-\psi|m} + \sqrt{\left(\frac{1+\alpha}{1-\alpha}\right)} P_{|\varphi-\psi|m}\right)\right],$$

(14) $$\mu_{\alpha m} = -\frac{\varrho_P \vartheta^2\pi^2 m^2}{p_{0m} b^3}\left\{\alpha\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \sqrt{(1-\alpha^2)}\left(-A_m N_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} M_{\varphi m}\right) + \alpha\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) + \sqrt{(1-\alpha)^2}\left(C_m P_{\varphi m} - \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} O_{\varphi m}\right) + \bar{C}_m\left[\sqrt{\left(\frac{1+\alpha}{1-\alpha}\right)} P_{|\varphi-\psi|m} - O_{|\varphi-\psi|m}\right]\right\}.$$

The symbols M, N, O, P and $\bar{C}$ have again the same significance as before.

If $\alpha = 0$, the shear force is given by the same expression as in the foregoing case since the twisting moments are zero.

The values of coefficients K_{01} and μ_{01} are given (using again the same analogy according to which a cross beam of differential width is considered as a beam on an elastic foundation [4]) by the following expressions:

$$(15)\qquad K_{01} = 2\lambda b \frac{1}{\mathrm{sh}^2\, 2\lambda b - \sin^2 2\lambda b} \{[2\,\mathrm{ch}\,\lambda(b \pm y) \cos \lambda(b \pm y)]\, S_e +$$

$$+ [\mathrm{ch}\,\lambda(b \pm y) \sin \lambda(b \pm y) + \mathrm{sh}\,\lambda(b \pm y) \cos \lambda(b \pm y)]\, T_e\},$$

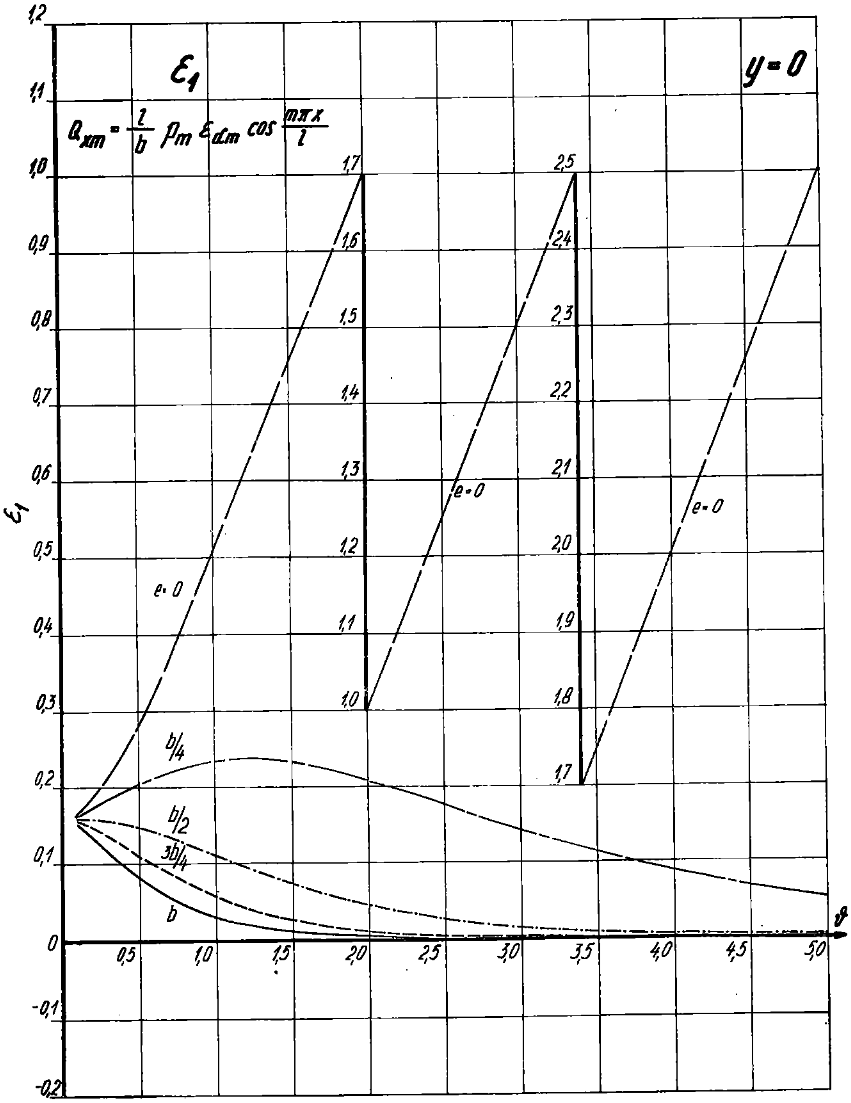

Fig. 7.

$$\mu_{01} = \frac{1}{\pi\sqrt{2\vartheta}}\,\frac{1}{\operatorname{sh}^2 2\lambda b - \sin^2 2\lambda b}\,\{[2 \operatorname{sh} \lambda(b \pm y) \sin \lambda(b \pm y)\, S_e + [\operatorname{ch} \lambda(b \pm y) \sin \lambda(b \pm y) - \operatorname{sh} \lambda(b \pm y) \cos \lambda(b \pm y)]\, T_e\}\,, \tag{16}$$

where S_e, T_e and λ have the significance given by eqs. (6). The upper signs in parentheses hold for $e \geqq y$, the lower ones for $e \leqq y$. The coefficients K_{0m} and μ_{0m} are obtained from eqs. (15) and (16) by substituting $m\vartheta$ for ϑ.

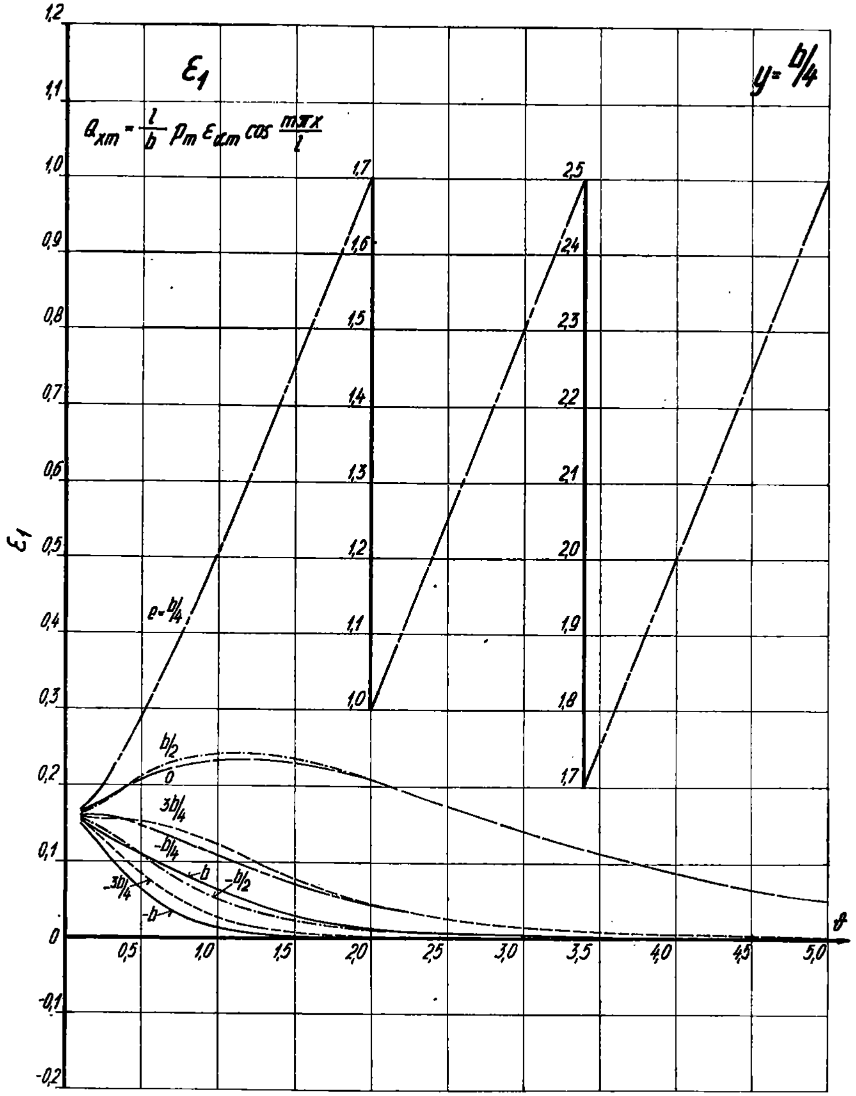

Fig. 8.

For $\alpha = 1$ the values of coefficients K_{11} and μ_{11}, using Guyon's results for isotropic plates, are obtained in the form:

$$K_{11} = \frac{\sigma}{2\,\mathrm{sh}^2\,\sigma}\{(\sigma\,\mathrm{ch}\,\sigma + \mathrm{sh}\,\sigma)\,\mathrm{ch}\,\vartheta\chi - \vartheta\chi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\chi + [(\sigma\,\mathrm{ch}\,\sigma - \mathrm{sh}\,\sigma)\,\mathrm{ch}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\varphi]\,U_\psi + [(2\,\mathrm{sh}\,\sigma + \sigma\,\mathrm{ch}\,\sigma)\,\mathrm{sh}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\varphi]\,V_\psi\}, \tag{17}$$

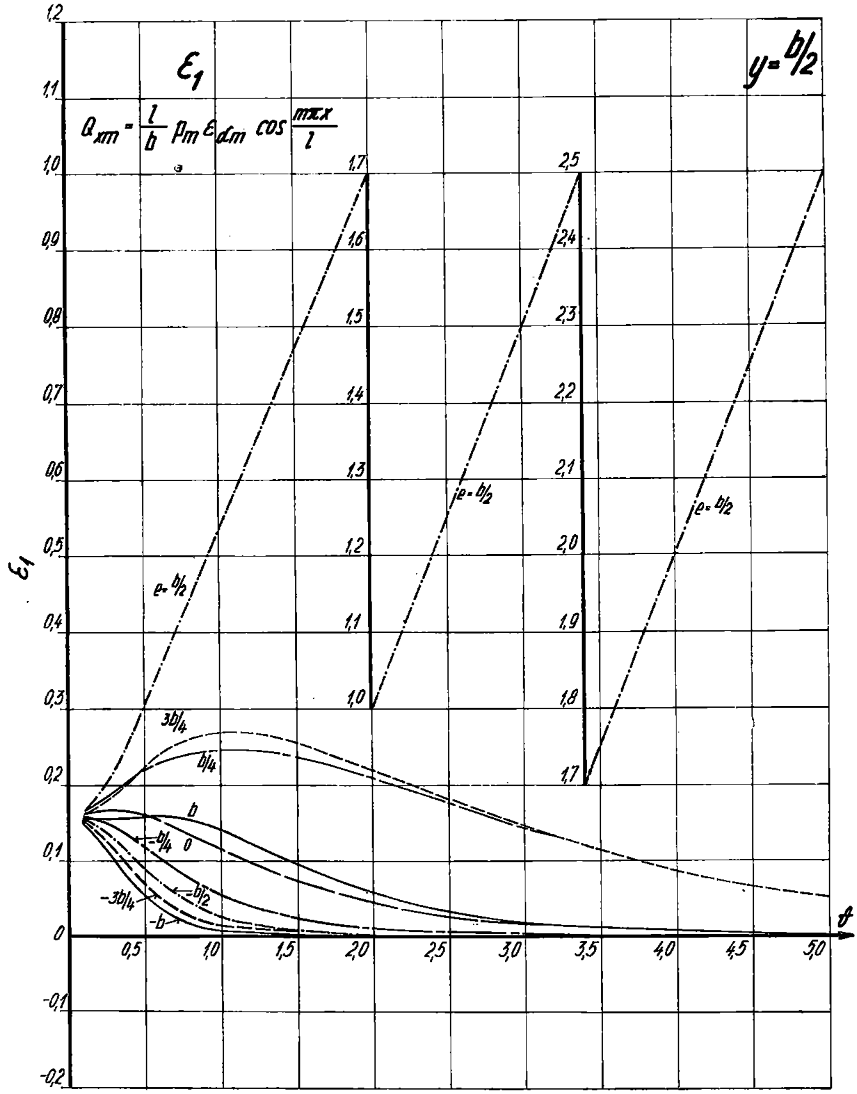

Fig. 9.

$$\mu_{11} = -\frac{1}{4\sigma\,\mathrm{sh}^2\,\sigma}\{(\sigma\,\mathrm{ch}\,\sigma - \mathrm{sh}\,\sigma)\,\mathrm{ch}\,\vartheta\chi - \vartheta\chi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\chi +$$
$$+ [(\sigma\,\mathrm{ch}\,\sigma - 3\,\mathrm{sh}\,\sigma)\,\mathrm{ch}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\varphi]\,U_\psi +$$
$$+ [\sigma\,\mathrm{ch}\,\sigma\,\mathrm{sh}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\varphi]\,V_\psi\,. \tag{18}$$

The symbols U_ψ, V_ψ, σ, χ are given by the relations (9). Coefficients K_{1m} and μ_{1m} are obtained from eqs. (17) and (18) by substituting $m\vartheta$ for ϑ.

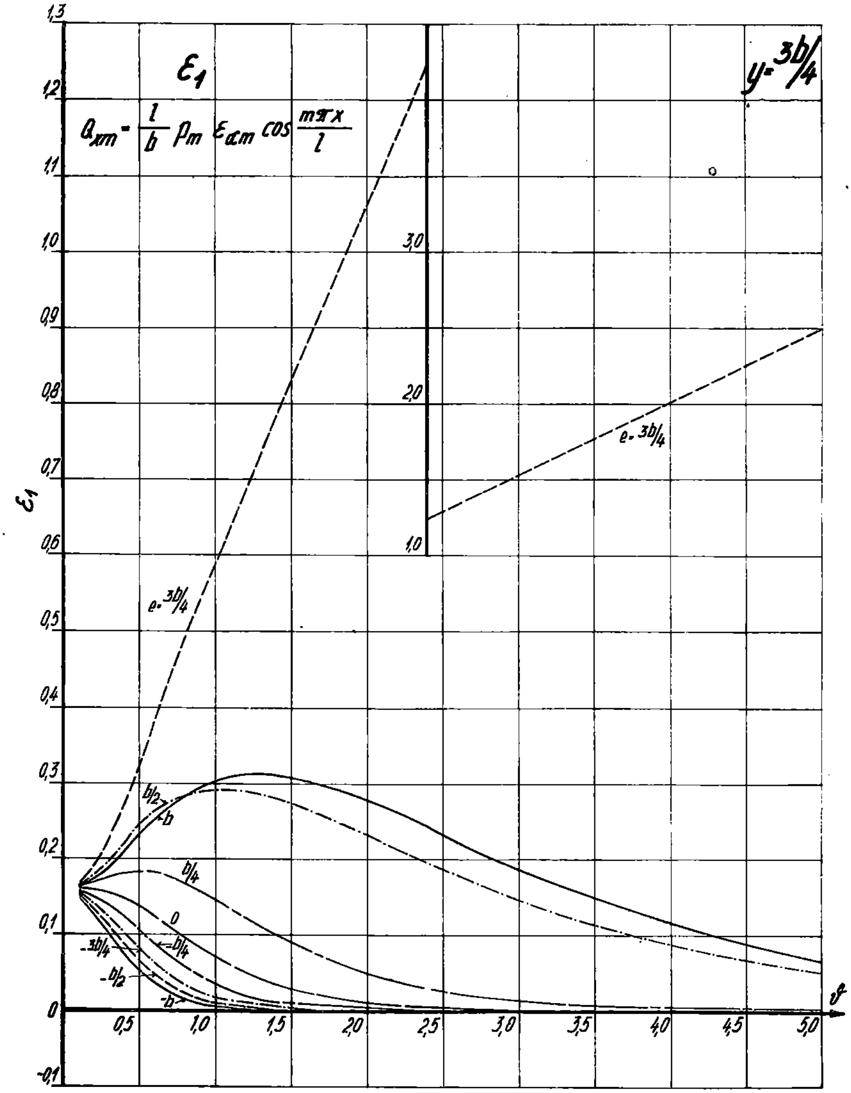

Fig. 10.

For the determination of shear force Q_x for a general value of α the coefficients $K_{\alpha m}$ and $\mu_{\alpha m}$ are calculated in terms of K_{0m}, μ_{0m} and K_{1m}, μ_{1m} using the interpolation formulae

(19)
$$K_{\alpha m} = K_{0m} + (K_{1m} - K_{0m})\sqrt{\alpha}\,,$$
$$\mu_{\alpha m} = \mu_{0m} + (\mu_{1m} - \mu_{0m})\sqrt{\alpha}\,.$$

For the determination of maximum values of longitudinal shear forces it is necessary to find the most effective position of loading in cross direction by means of

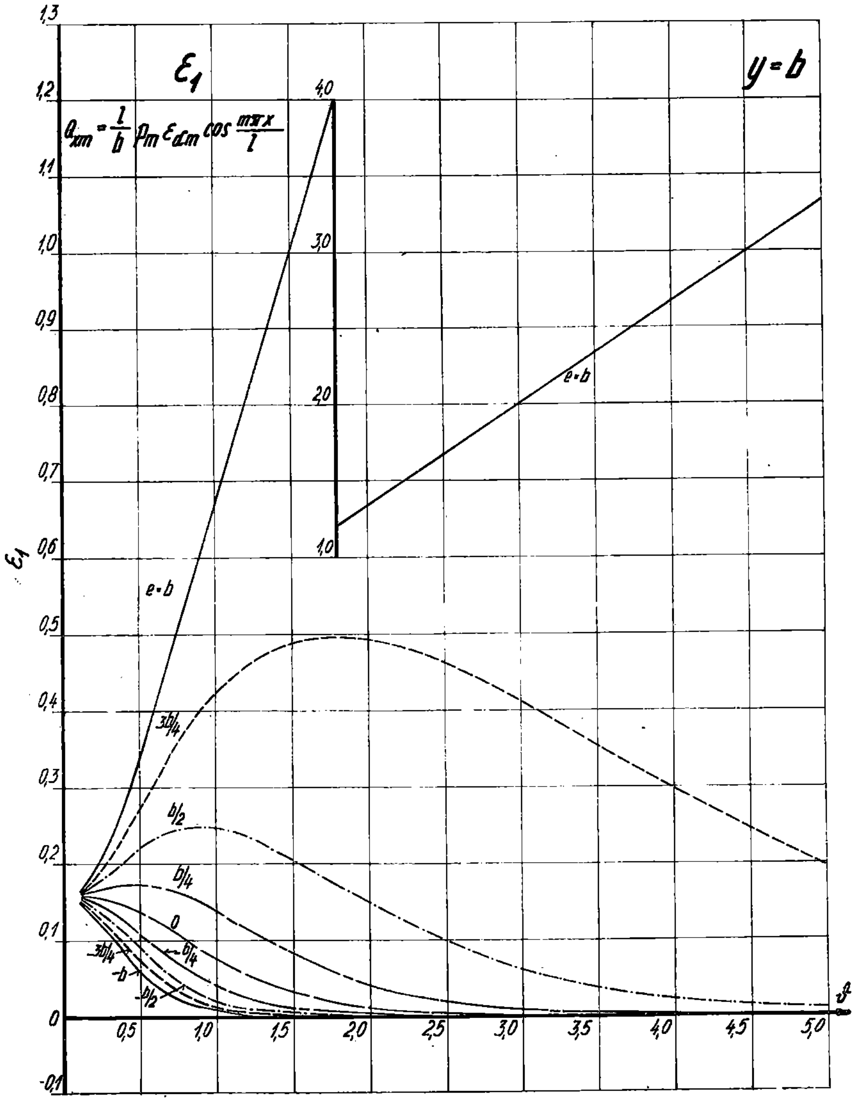

Fig. 11.

influence lines ε (or K and μ in case of grillages) and in longitudinal direction in the usual way according to the rules of structural mechanics by means of influence lines of a simple beam.[1])

2.3 Cross shear forces in a plate

The shear force in direction Y per unit length is given by the expression

$$Q_y = -\varrho_P \frac{\partial^3 w}{\partial y^3} - 2\gamma \frac{\partial^3 w}{\partial x^2\, \partial y} = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x}\,. \tag{20}$$

Introducing $2\gamma = \alpha\sqrt{(\varrho_T \varrho_P)}$, as in the foregoing case, we obtain after differentiation and rearrangement the following expression for the shear force in cross direction in a form suitable for practical use

$$Q_y = \sum_{m=1,2,3\ldots} v_{\alpha m} p_{0m} \sin \frac{m\pi x}{l}\,, \tag{21}$$

where

$$v_{\alpha m} = \frac{m^3\pi^3\vartheta^3\varrho_P}{b^3 p_{0m}} \Bigg[(1-\alpha)\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) - \tag{22}$$

$$- (1+\alpha)\sqrt{\left(\frac{1-\alpha}{2}\right)}\left(-A_m N_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} M_{\varphi m}\right) -$$

$$- (1-\alpha)\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) -$$

$$- (1+\alpha)\sqrt{\left(\frac{1-\alpha}{2}\right)}\left(-C_m P_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} O_{\varphi m}\right) \pm \sqrt{[2(1+\alpha)]}\, \bar{C}_m O_{|\varphi-\psi|m}\Bigg]\,.$$

The symbols M, N, O, P, $\bar{C}$ have the same significance as before; the positive sign

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1) Shear forces in a single beam of small or lightly loaded structures may be approximately determined by using the expression

$$Q_x = \frac{\partial M_x}{\partial x} = \frac{\partial (K_{\alpha m} M_{0x})}{\partial x} = \sum_{m=1,2,3\ldots} K_{\alpha m} \frac{p_{0m} l}{2bm\pi} \cos \frac{m\pi x}{l}\,.$$

It is therefore possible to use even for approximate determination of longitudinal shear force in beams the coefficient of cross distribution $K_{\alpha m}$, the value of which is given by formula (13) or formulae (15), (17) and (19).

of the last term of equation (22) holds for the case when $\psi \geqq \varphi$, the negative one for the case when $\psi \leqq \varphi$.

If $\alpha = 0$, the shear force Q_y may be expressed by the equation

$$Q_y = \sum_{m=1,2,3\ldots} v_{0m} p'_{om} \sin \frac{m\pi x}{l}, \tag{21a}$$

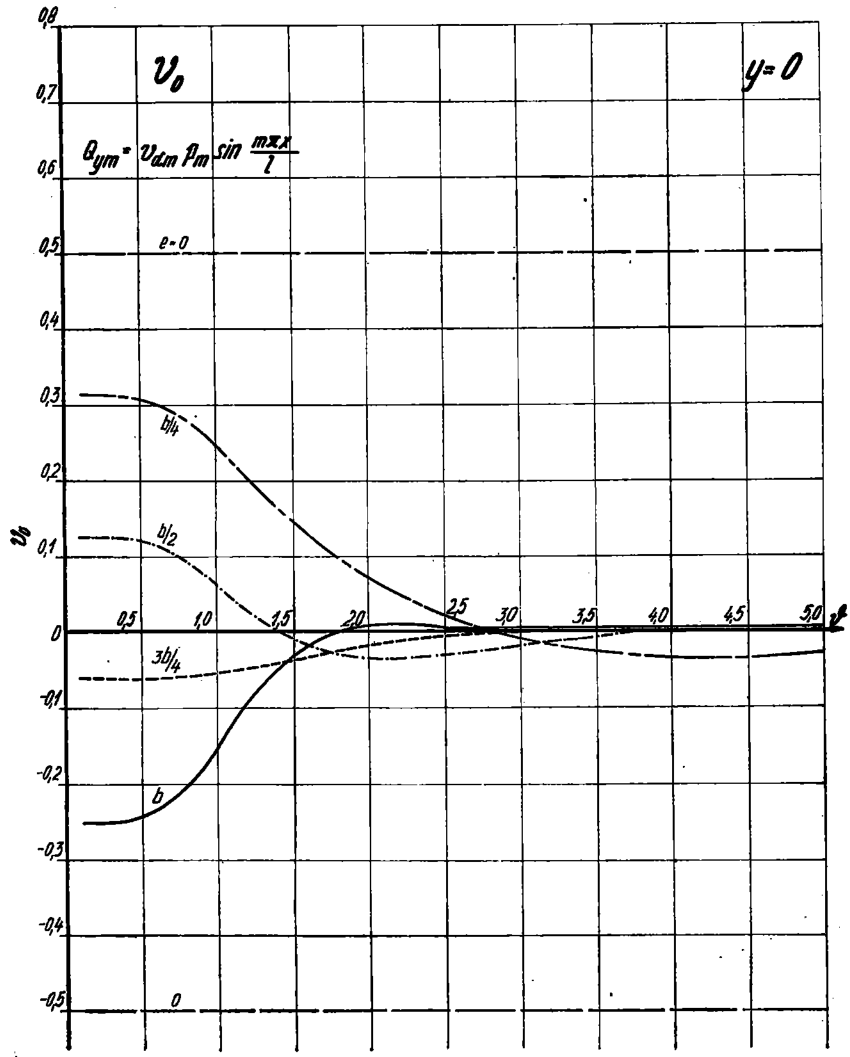

Fig. 12.

where the coefficient v_{01} is given by

(23) $$v_{01} = \pm \frac{1}{\operatorname{sh}^2 2\lambda b - \sin^2 2\lambda b} \{[\operatorname{ch} \lambda(b \pm y) \sin \lambda(b \pm y) +$$

$$+ \operatorname{sh} \lambda(b \pm y) \cos \lambda(b \pm y)] S_e + [\operatorname{sh} \lambda(b \pm y) \sin \lambda(b \pm y)] T_e\} .$$

The symbols S_e, T_e and λ are given by formulae (6). The value of v_{0m} is obtained from equation (23) by substitution of $m\vartheta$ for ϑ. The upper sign holds for the case when $e \geqq y$, the lower one for the case when $e \leqq y$.

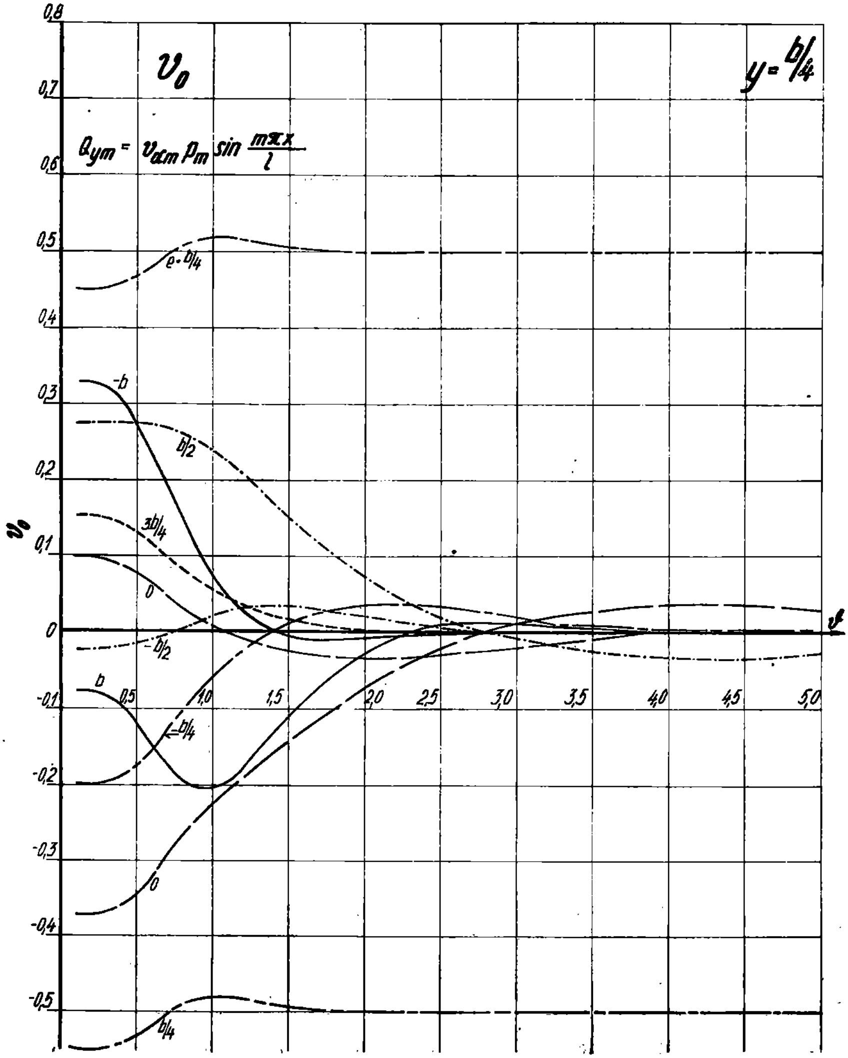

Fig. 13.

The coefficient ν_{11} for the case when $\alpha = 1$ is calculated using a similar procedure as above

(24) $$\nu_{11} = \frac{1}{4\,\mathrm{sh}^2\,\sigma}\{\pm 2\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\chi + 2\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\varphi U_\psi + 2\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\varphi V_\psi\}$$

and the shear force is expressed by eq. (21a) in which the index 0 of coefficient ν was replaced by 1. The significance of coefficients φ, ψ, σ, χ, U_ψ, V_φ was given before. The positive sign of the first term of eq. (24) corresponds to the case when $\psi \geqq \varphi$, the negative one to the case when $\psi \leqq \varphi$. The value of ν_{1m} is obtained from eq. (24) by substitution of $m\vartheta$ for ϑ.

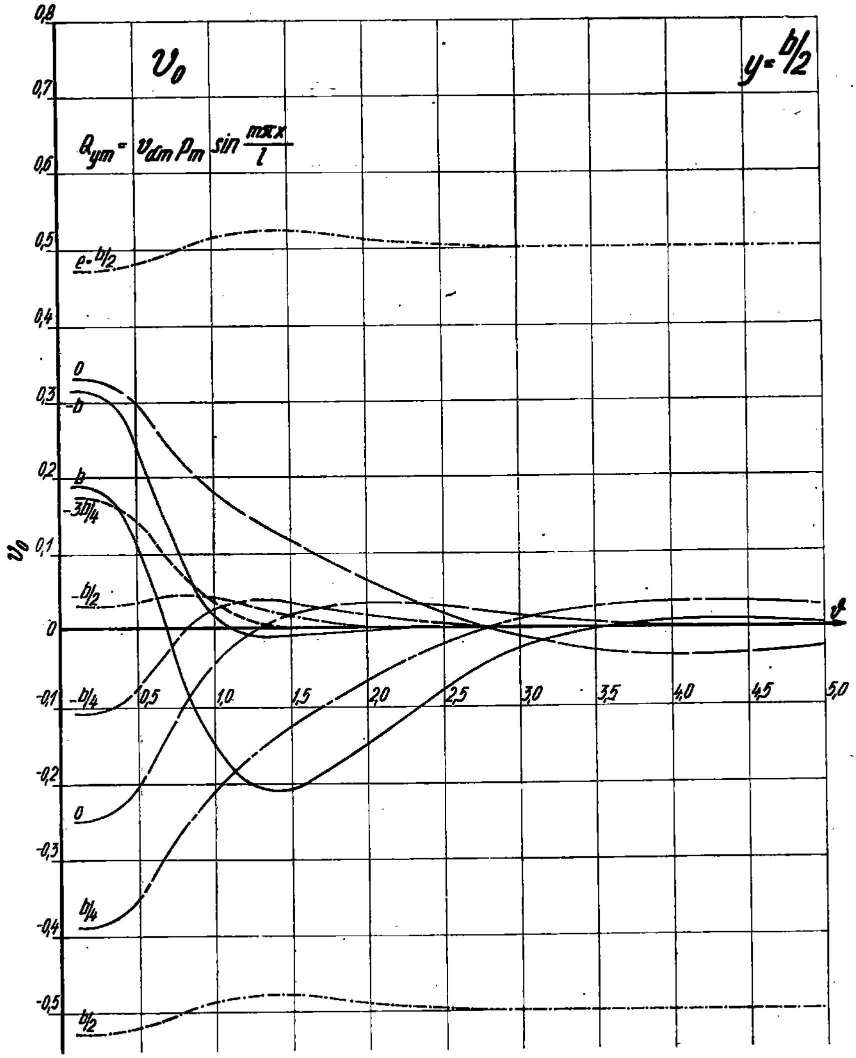

Fig. 14.

Values of coefficients ν_α for various values of α and $\vartheta = 0{,}66874$ were calculated by using eq. (22) [2]. It was proved by analysis of the variation of these coefficients in dependence on α that the interpolation formulae may be used in full extent for the determination of a general value of $\nu_{\alpha m}$ in terms of ν_{0m} and ν_{1m} in the form

$$\nu_{\alpha m} = \nu_{0m} + (\nu_{1m} - \nu_{0m})\sqrt{\alpha}\,. \tag{25}$$

Numerical values of the coefficients ν_{0m} and ν_{1m} are given by the diagrams in fig. 12–19.

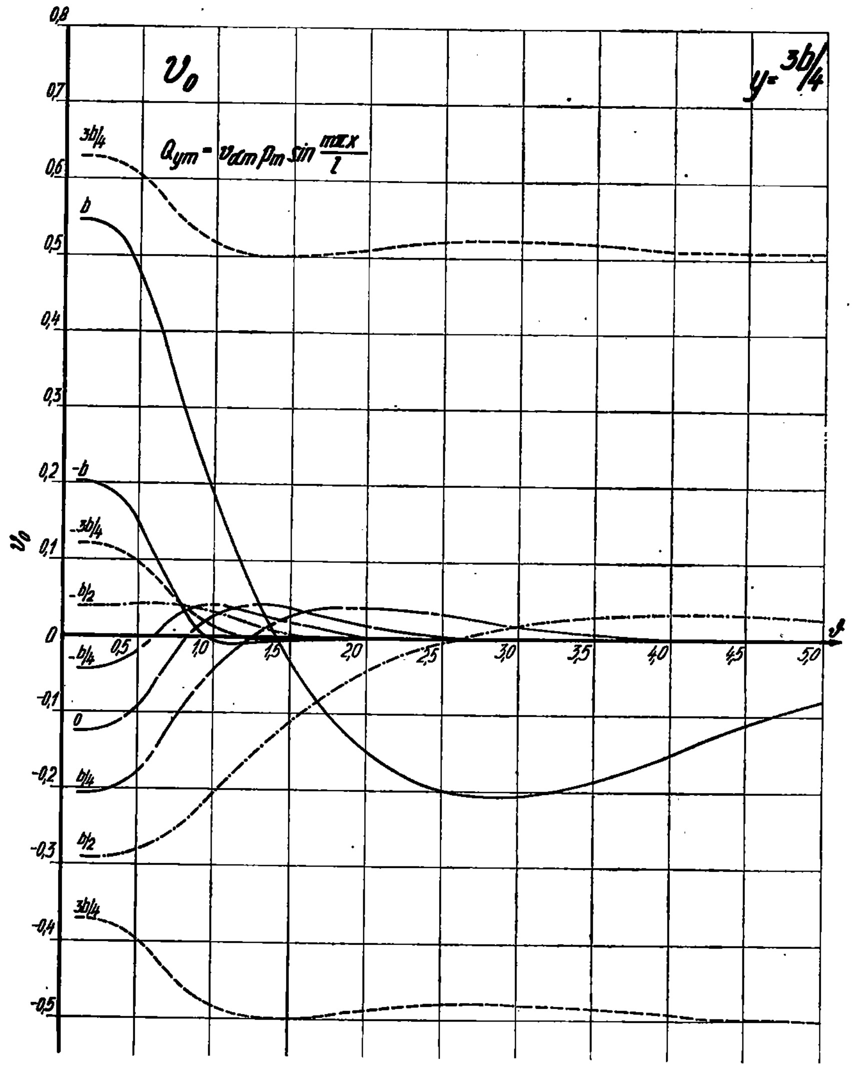

Fig. 15.

2.4 Shear forces in cross-beams in a grillage

The shear force in direction Y per unit width is given by the expression

$$Q_y = -\varrho_P \frac{\partial^3 w}{\partial y^3} - \gamma_T \frac{\partial^3 w}{\partial x^2\,\partial y} = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x}. \tag{26}$$

If we express the shear force in the usual form we obtain

$$Q_y = \sum_{m=1,2,3\ldots} p_{0m} \sin\frac{m\pi x}{l}\left[\kappa_{\alpha m} + \frac{m\pi b}{l}\,\frac{2\gamma_T}{\gamma_T+\gamma_P}\,\tau_{\alpha m}\right], \tag{27}$$

where the coefficient $\kappa_{\alpha m}$ has the value

$$\kappa_{\alpha m} = -\frac{m^3\pi^3\vartheta^3\varrho_P}{b^3 p_{0m}}\left[(2\alpha-1)\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \right. \tag{28}$$

$$+ (2\alpha+1)\sqrt{\left(\frac{1-\alpha}{2}\right)}\left(-A_m N_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} M_{\varphi m}\right) -$$

$$-(2\alpha-1)\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) +$$

$$+ (2\alpha+1)\sqrt{\left(\frac{1-\alpha}{2}\right)}\left(-C_m P_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} O_{\varphi m}\right) \pm$$

$$\left.\pm \frac{\bar{C}_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}}\left(-\alpha P_{|\varphi-\psi|m} + \sqrt{(1-\alpha^2)}\,O_{|\varphi-|\psi m}\right)\right]$$

and the coefficient $\tau_{\alpha m}$, used for computation of twisting moments, the value

$$\tau_{\alpha m} = \alpha\sqrt{(\varrho_T\varrho_P)}\,\frac{m^2\pi^2\vartheta}{b^2 l p_{0m}}\left[\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \right. \tag{29}$$

$$+ \sqrt{\left(\frac{1-\alpha}{2}\right)}\left(-A_m N_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} M_{\varphi m}\right) -$$

$$-\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) +$$

$$+\sqrt{\left(\frac{1-\alpha}{2}\right)}\left(-C_m P_{\varphi m}+\frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}}O_{\varphi m}\right)\pm \bar{C}_m\frac{P_{|\varphi-\psi|m}}{\sqrt{\left(\frac{1-\alpha}{2}\right)}}\Bigg],$$

where the symbols M, N, O, P, $\bar{C}$ have the same values as before.

If $\alpha = 0$, the shear force is given by the same expression (23) as in the foregoing case, since the twisting moments are zero. Therefore, for $\alpha = 0$ we have $\kappa_{0m} = \nu_{0m}$.

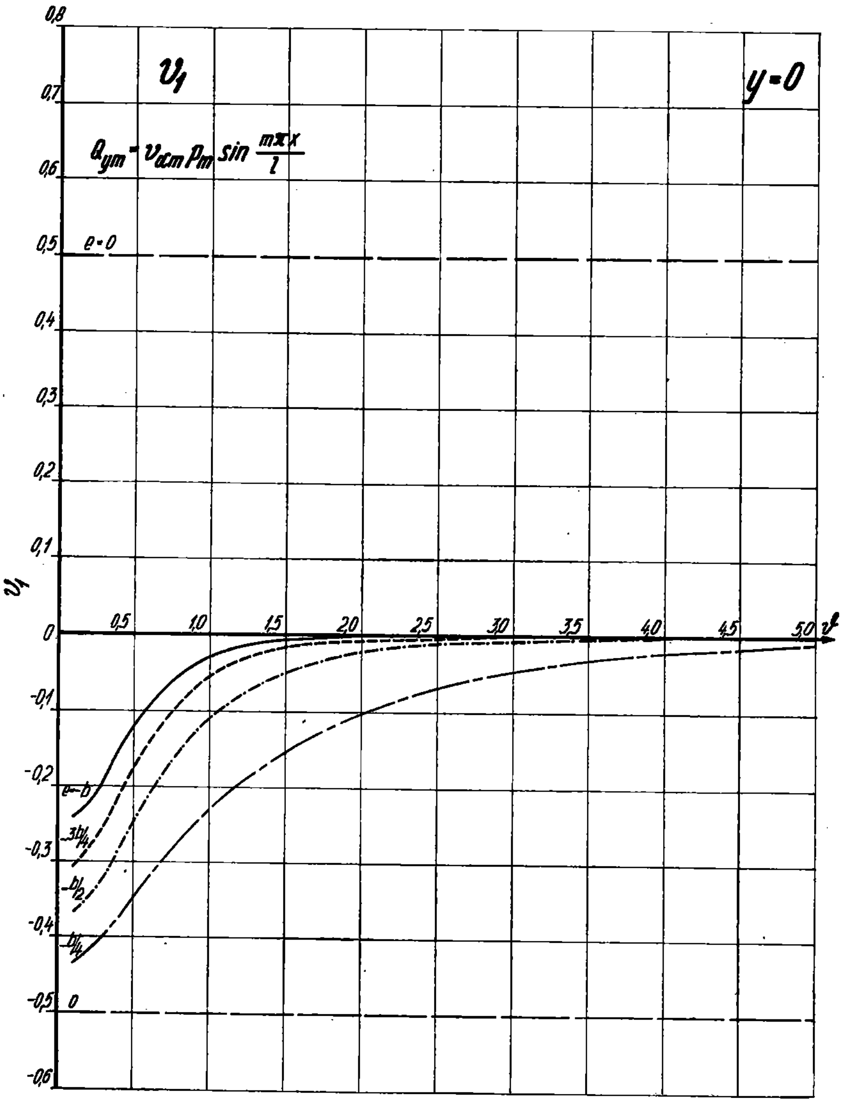

Fig. 16.

For $\alpha = 1$, using again the usual form of expression for shear force [formula (27) in which the indexes α of coefficients κ and τ were replaced by 1], the values of coefficients κ_{11} and τ_{11} are given by the expressions

$$\kappa_{11} = -\frac{1}{4\,\mathrm{sh}^2\,\sigma}\{\pm[(\sigma\,\mathrm{ch}\,\sigma - 2\,\mathrm{sh}\,\sigma)\,\mathrm{sh}\,\vartheta\chi - \vartheta\chi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\chi] + [(\sigma\,\mathrm{ch}\,\sigma - 4\,\mathrm{sh}\,\sigma)\,\mathrm{sh}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\varphi]\,U_\psi + [(\sigma\,\mathrm{ch}\,\sigma - \mathrm{sh}\,\sigma)\,\mathrm{ch}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\varphi]\,V_\psi\}, \tag{30}$$

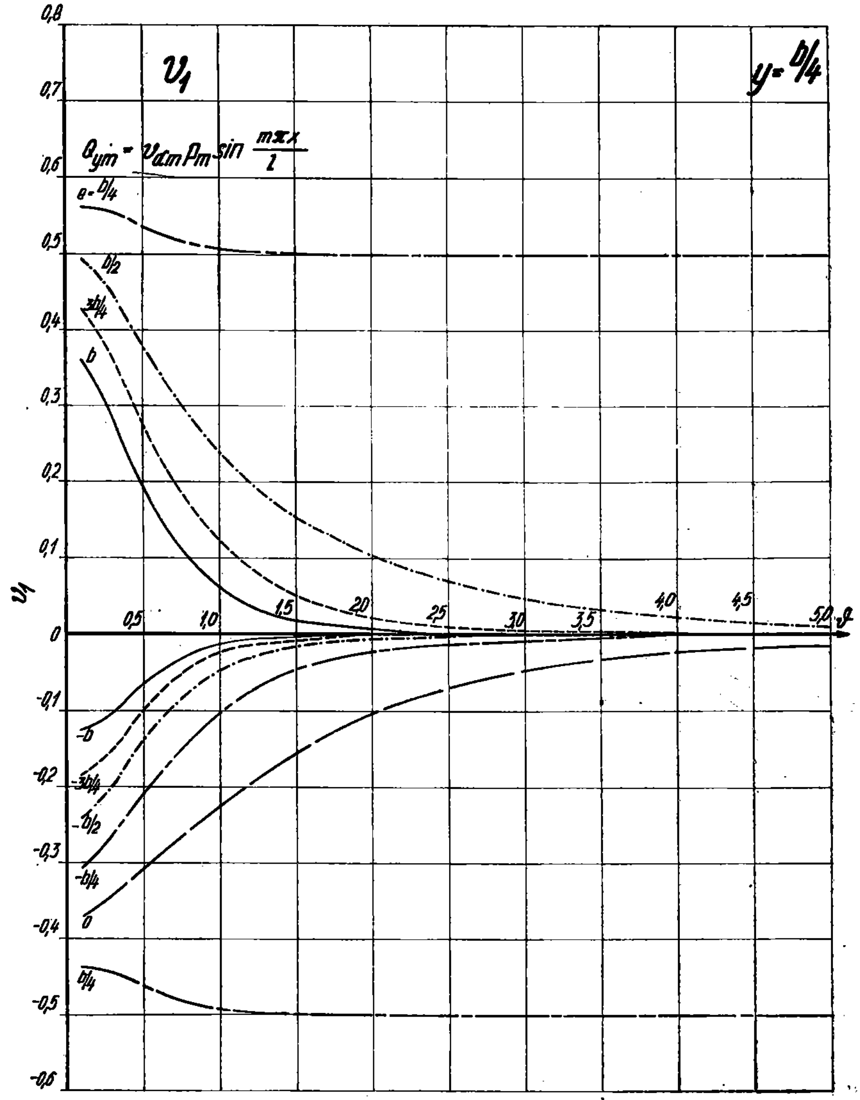

Fig. 17.

(31) $$\tau_{11} = \frac{l}{4b\pi m}\frac{1}{\operatorname{sh}^2\sigma}\{\pm(\sigma\operatorname{ch}\sigma\operatorname{sh}\vartheta\chi - \vartheta\chi\operatorname{sh}\sigma\operatorname{ch}\vartheta\chi) + $$
$$+ [(\sigma\operatorname{ch}\sigma - 2\operatorname{sh}\sigma)\operatorname{sh}\vartheta\varphi - \vartheta\varphi\operatorname{sh}\sigma\operatorname{ch}\vartheta\varphi]\,U_\psi +$$
$$+ [(\operatorname{sh}\sigma + \sigma\operatorname{ch}\sigma)\operatorname{ch}\vartheta\varphi - \vartheta\varphi\operatorname{sh}\sigma\operatorname{sh}\vartheta\varphi]\,V_\psi]\}.$$

The positive sign of the first terms of the equations holds for the case when $\psi \geqq \varphi$, the negative one for the case when $\psi \leqq \varphi$.

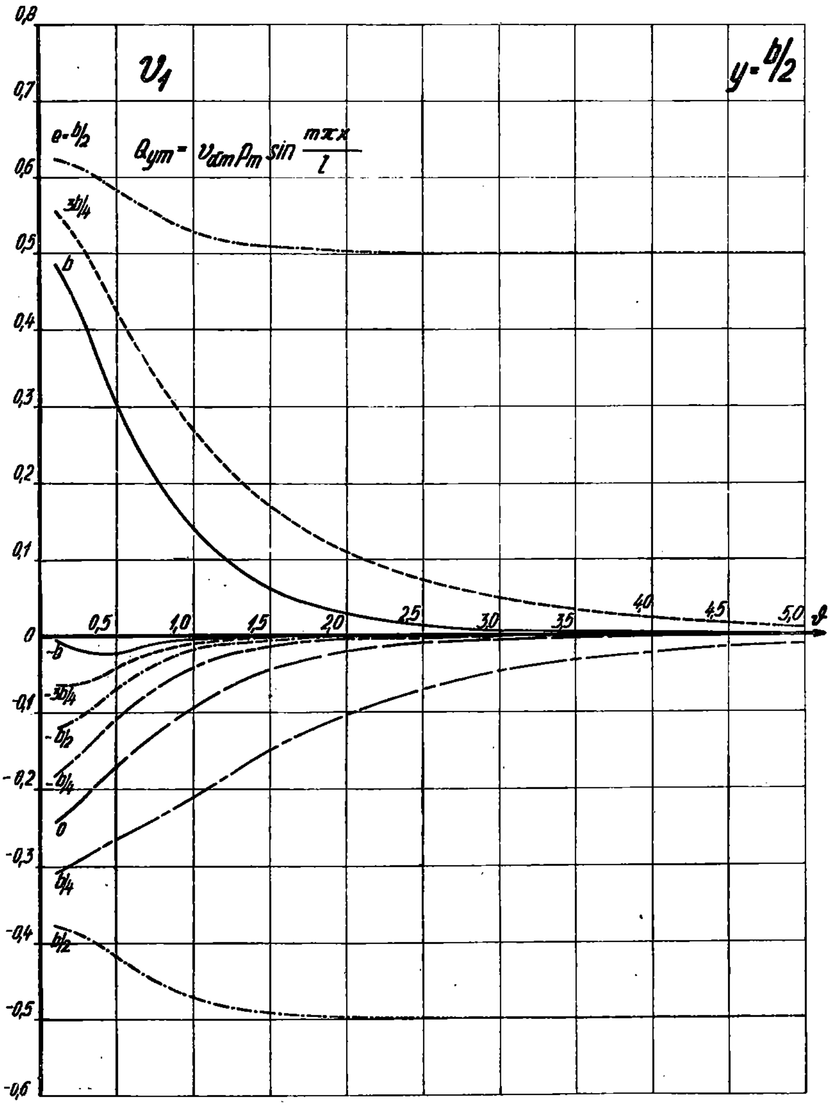

Fig. 18.

The used symbols have the same significance as before. κ_{1m} and τ_{1m} are obtained from these expressions by substituting $m\vartheta$ for ϑ.

To determine the shear force Q_y for a general value of α the coefficients $\kappa_{\alpha m}$ and $\tau_{\alpha m}$ are calculated in terms of $\kappa_{0m} = \nu_{0m}$, κ_{1m} and τ_{1m} by means of the following interpolation formulae:

(32)
$$\kappa_{\alpha m} = \kappa_{0m} + (\kappa_{1m} - \kappa_{0m}) \sqrt{(\alpha)},$$
$$\tau_{\alpha m} = \tau_{1m} \sqrt{(\alpha)}.$$

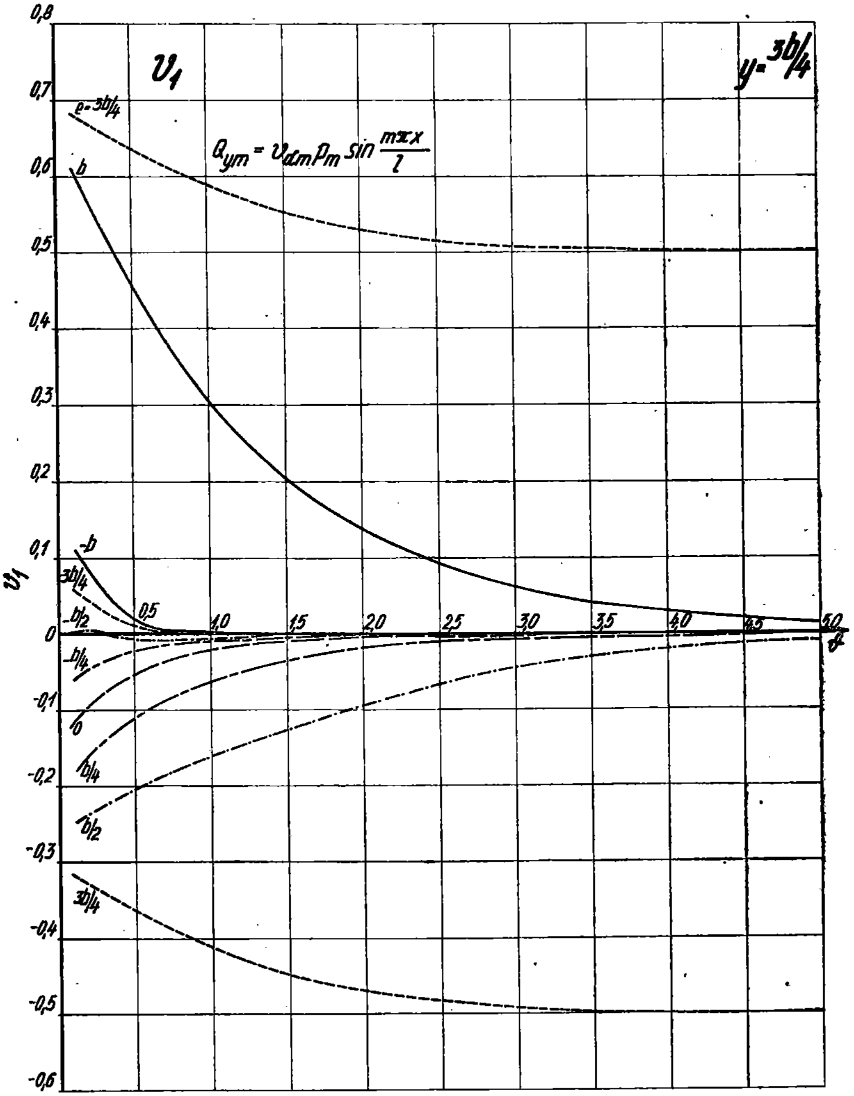

Fig. 19.

The values of coefficient κ_{1m} are given by the diagrams in fig. 20–23.

For the determination of maximum values of cross shear forces is decisive the most effective position of loading found in cross direction by means of influence lines ν (or κ and τ in case of grillages), in longitudinal direction usually at the place of loading.[2]

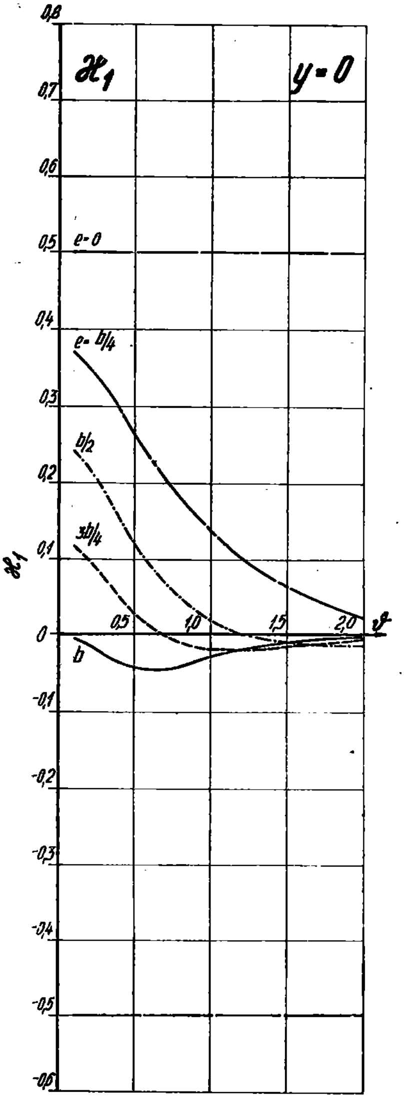

Fig. 20.

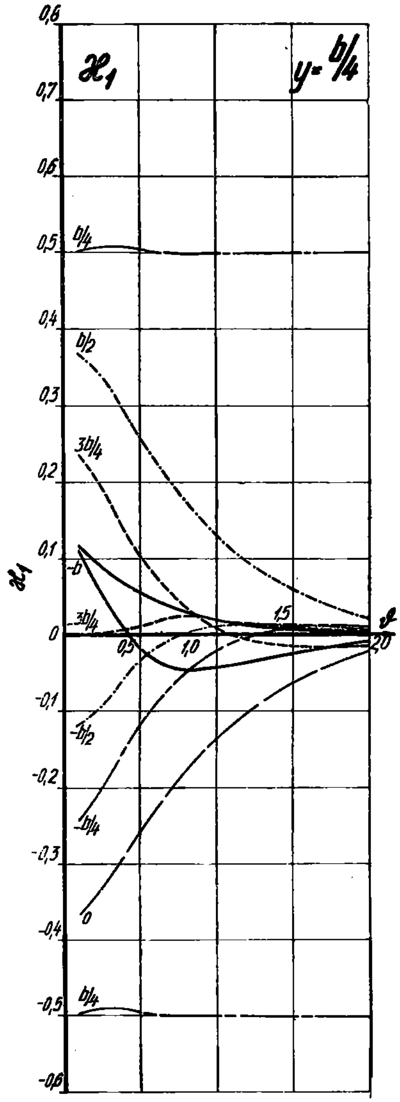

Fig. 21.

At boundaries $x = 0; l$, $y = \pm b$ the reactione $\bar{Q}_x$ and $\bar{Q}_y$ have on account of twisting moments replacement by additional forces a different value than was obtained from the foregoing relations for shear forces.

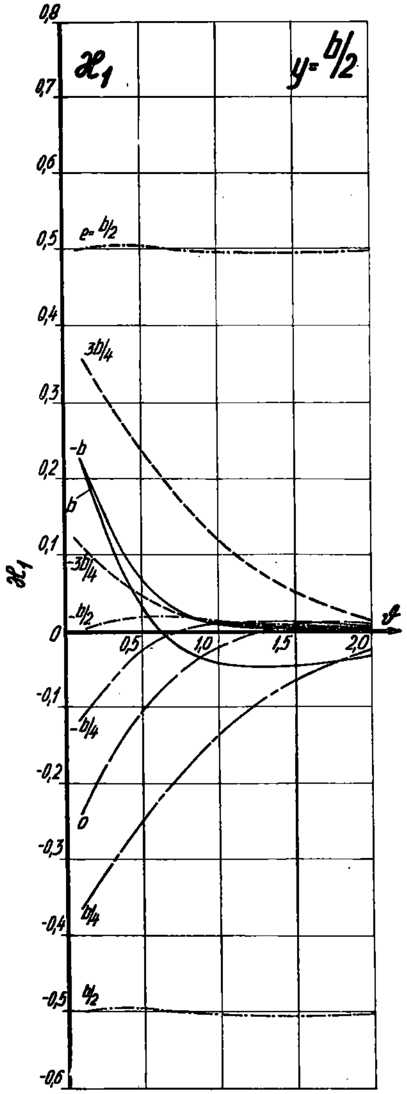

Fig. 22.

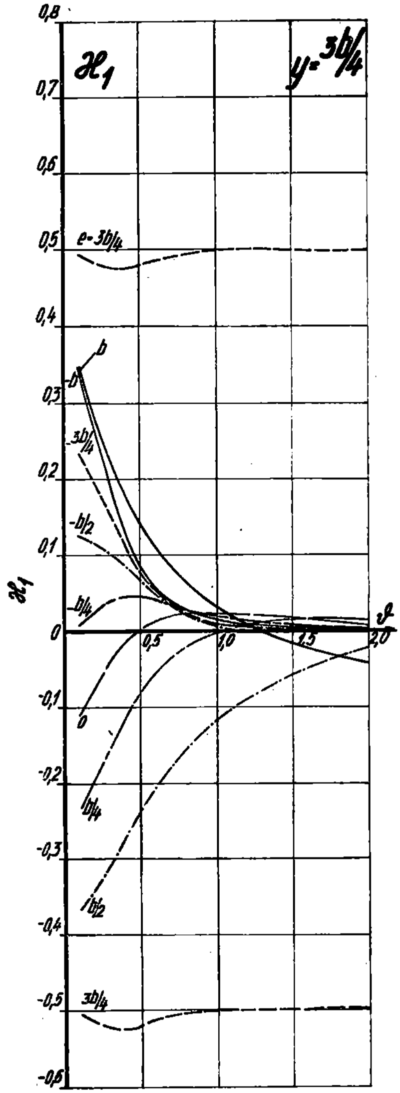

Fig. 23.

3.1 Reaction in longitudinal direction in a plate

Reaction $\bar{Q}_x$ in longitudinal direction per unit width in a plate at boundaries $x = 0; l$ is given by the relation

$$(33) \qquad \bar{Q}_x = -\varrho_T \frac{\partial^3 w}{\partial x^3} - 4\gamma \frac{\partial^3 w}{\partial x\, \partial y^2} = \frac{\partial}{\partial x}\left(M_x + \frac{4\gamma}{\varrho_P} M_y\right).$$

Analogously to eq. (3) we obtain

$$(34) \qquad \bar{\varepsilon}_{\alpha m} = \frac{m^3 \varrho_T \pi^3 b}{l^4 p_{0m}} \left\{ (1 - 2\alpha^2)\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \right.$$

$$+ 2\alpha \sqrt{(1 - \alpha^2)}\left(A_m N_{\varphi m} - \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) +$$

$$+ (1 - 2\alpha^2)\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) +$$

$$+ 2\alpha \sqrt{(1 - \alpha^2)}\left(- C_m P_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} O_{\varphi m}\right) +$$

$$\left. + \bar{C}_m \left[(1 + 2\alpha)\, O_{|\varphi - \psi| m} + \sqrt{\left(\frac{1+\alpha}{1-\alpha}\right)} P_{|\varphi - \psi| m}\right] \right\},$$

if the reaction at boundaries $x = 0; l$ is given by

$$(35) \qquad \bar{Q}_x = \sum_{m=1,2,3\ldots} p_{0m} \bar{\varepsilon}_{\alpha m} \frac{l}{b} \cos \frac{m\pi x}{l}.$$

---

[2]) Shear forces in cross-beams of small or lightly loaded structures may be found, similarly as in longitudinal direction, from the relation

$$Q_y = \frac{\partial M_y}{\partial y} = \sum_{m=1,2,3\ldots} \kappa_{\alpha m} p_{0m} \sin \frac{m\pi x}{l},$$

where the coefficient $\kappa_{\alpha m}$ is given by formula (28) or formulae (23), (30) and (32).

Similarly for $\alpha = 1$, eq. (8) turns into the form

$$\bar{\varepsilon}_{11} = \frac{\vartheta}{4 \operatorname{sh}^2 \sigma} \{(3 \operatorname{sh} \sigma - \sigma \operatorname{ch} \sigma) \operatorname{ch} \vartheta\chi + \vartheta\chi \operatorname{sh} \sigma \operatorname{sh} \vartheta\chi + \tag{36}$$

$$+ [(5 \operatorname{sh} \sigma - \sigma \operatorname{ch} \sigma) \operatorname{ch} \vartheta\varphi + \vartheta\varphi \operatorname{sh} \sigma \operatorname{sh} \vartheta\varphi] U_\psi +$$

$$+ [(2 \operatorname{sh} \sigma - \sigma \operatorname{ch} \sigma) \operatorname{sh} \vartheta\varphi + \vartheta\varphi \operatorname{sh} \sigma \operatorname{ch} \vartheta\varphi] V_\psi\} .$$

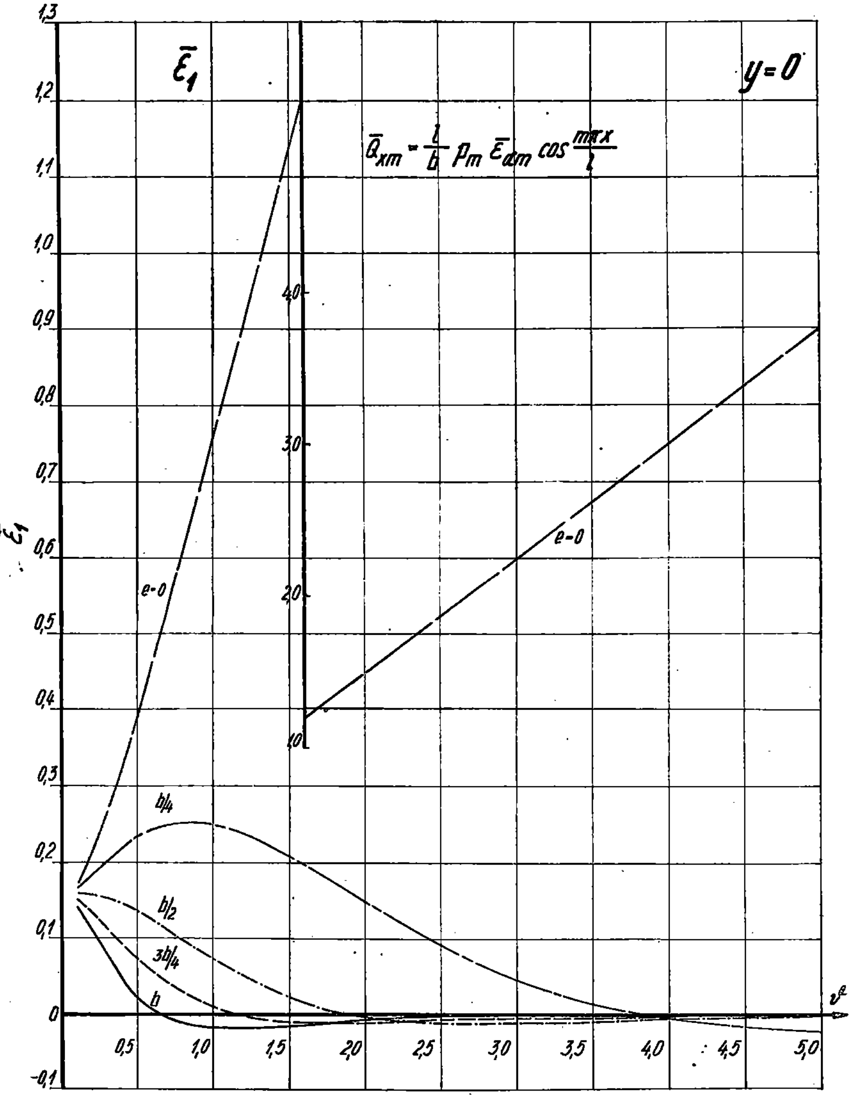

Fig. 24.

$\bar{\varepsilon}_{1m}$ is obtained from eq. (36) by substitution of $m\vartheta$ for ϑ and $m\sigma$ for σ.

The coefficient ε_0 remains the same also at the boundaries $x = 0; l$.

The coefficient $\bar{\varepsilon}_{\alpha m}$ for a general value of α is determined in terms of ε_{0m} and $\bar{\varepsilon}_{1m}$ using the interpolation formulae (10).

Numerical values of coefficients $\bar{\varepsilon}_{1m}$ are given by the diagrams in fig. 24–28.

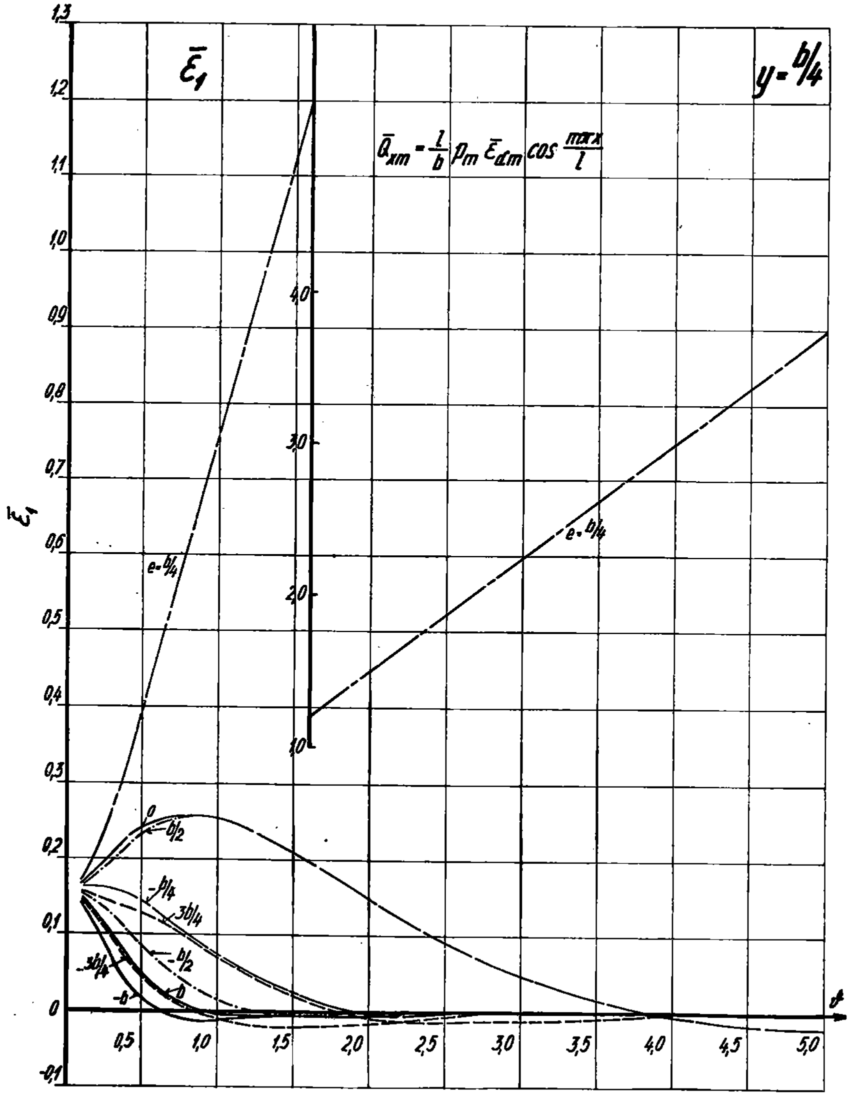

Fig. 25.

3.2 Reaction in longitudinal beams in a grillage

Analogously to the forgoing case the reaction in longitudinal beams in a grillage per unit width at boundaries $x = 0$; l is given by the expression

$$\bar{Q}_x = \sum_{m=1,2,3\ldots} p_{0m} \cos \frac{m\pi x}{l} \left(\frac{l}{2b\pi m} K_{\alpha m} + \frac{2\gamma_P}{\varrho_P} \frac{m\pi b}{l} \mu_{\alpha m} \right). \tag{37}$$

For $\alpha = 1$ holds the same formula, except that the coefficients have indexes 1 instead of α.

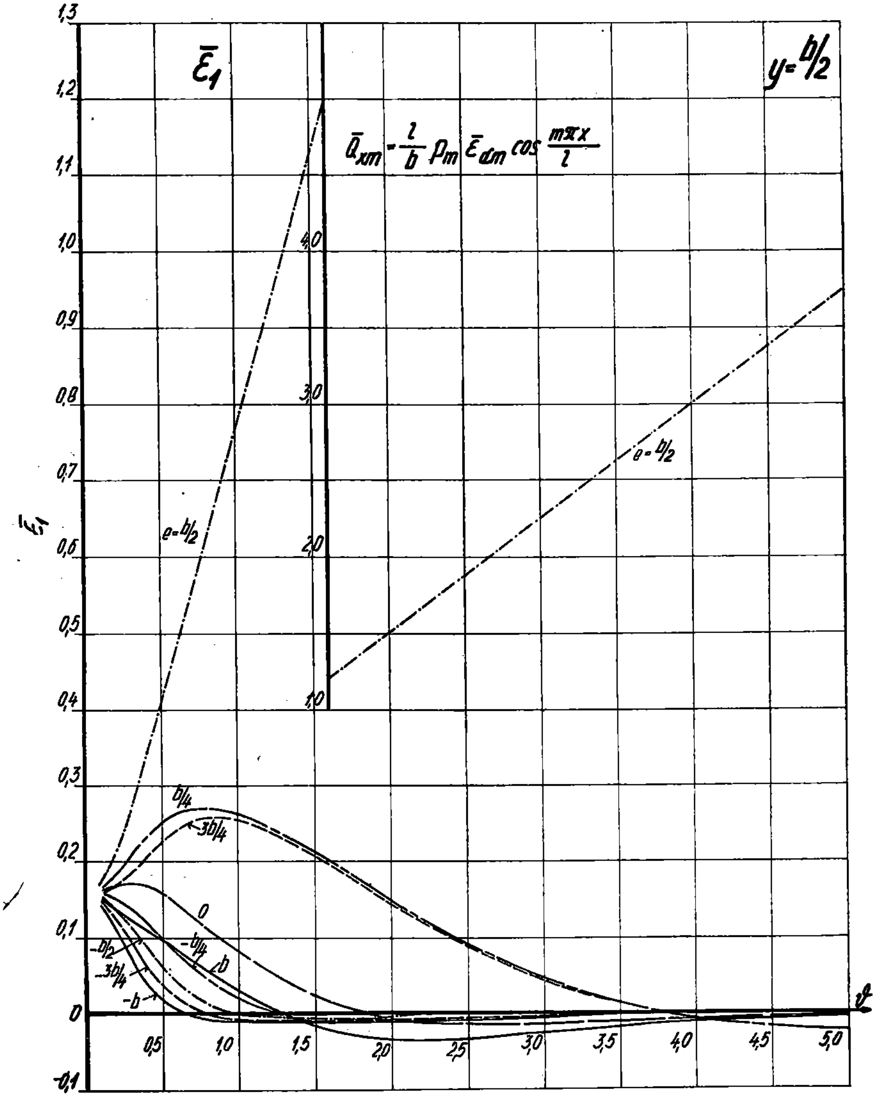

Fig. 26.

Expressions for K_{am} and μ_{am} are given by the relations (13) and (14) or may be obtained by means of the interpolation formulae using K_{0m}, K_{1m} and μ_{0m}, μ_{1m}.

3.3 Reaction in cross direction in a plate

At boundaries $y = \pm b$ the reaction $\bar{Q}_y$ has the value

$$\bar{Q}_y = -\varrho_P \frac{\partial^3 w}{\partial y^3} - 4\gamma \frac{\partial^3 w}{\partial x^2 \partial y} = \frac{\partial}{\partial y}\left(M_y + \frac{4\gamma}{\varrho_T} M_x\right). \tag{38}$$

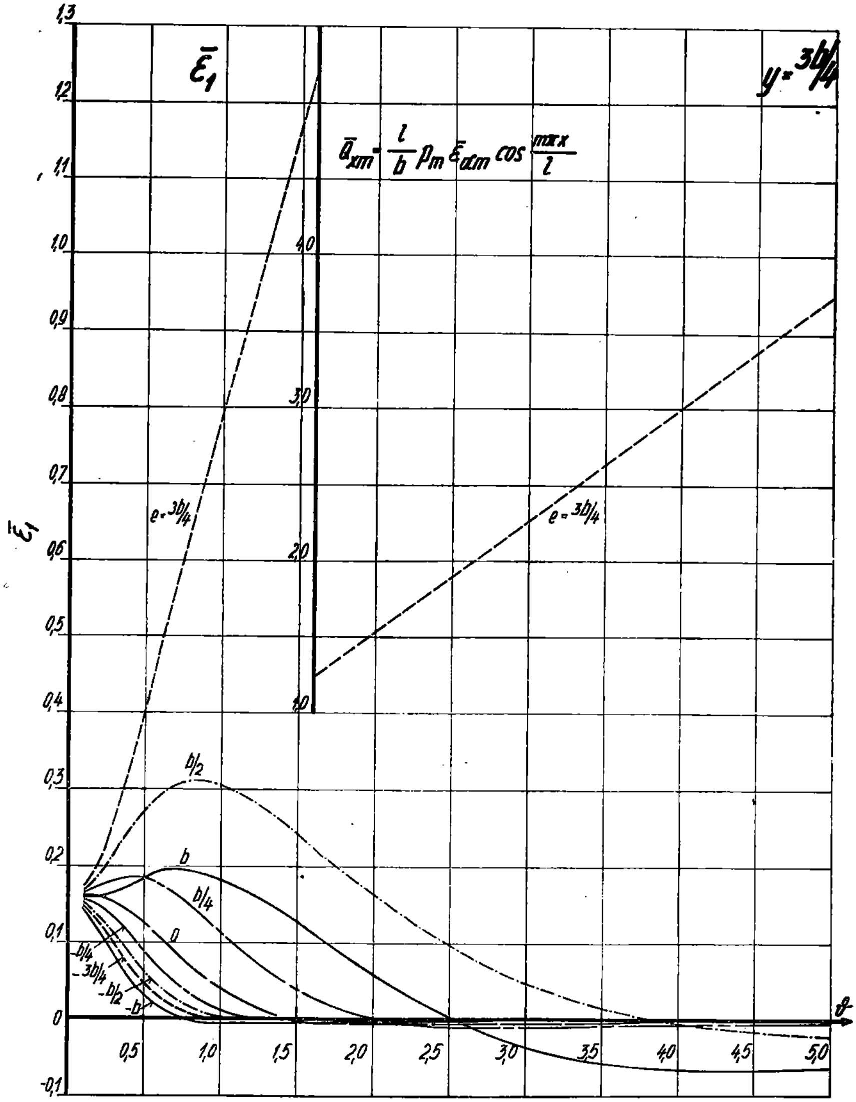

Fig. 27.

Expressing the reaction again in the form

$$(39)\qquad \bar{Q}_y = \sum_{m=1,2,3\ldots} p_{0m}\bar{v}_{\alpha m} \sin\frac{m\pi x}{l}$$

we have, using the same notation as before,

$$(40)\qquad \bar{v}_{\alpha m} = \frac{m^3\pi^3\vartheta^3\varrho_P}{b^3 p_{0m}}\left[\sqrt{\left(\frac{1+\alpha}{2}\right)}\left(A_m M_{\varphi m} + \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} N_{\varphi m}\right) + \right.$$

$$+ \sqrt{\left(\frac{1-\alpha}{2}\right)}\left(A_m N_{\varphi m} - \frac{B_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} M_{\varphi m}\right) -$$

$$- \sqrt{\left(\frac{1+\alpha}{2}\right)}\left(C_m O_{\varphi m} + \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} P_{\varphi m}\right) +$$

$$+ \sqrt{\left(\frac{1-\alpha}{2}\right)}\left(C_m P_{\varphi m} - \frac{D_m}{\sqrt{\left(\frac{1-\alpha}{2}\right)}} O_{\varphi m}\right) \mp$$

$$\left.\mp \left(\sqrt{[2(1+\alpha)]}\, \bar{C}_m O_{|\varphi-\psi|m} \mp \alpha\bar{C}_m \frac{P_{|\varphi-\psi|m}}{\sqrt{\left(\frac{1-\alpha}{2}\right)}}\right)\right],$$

$$(41)\qquad \bar{v}_{11} = \frac{1}{4\,\mathrm{sh}^2\sigma}\{\pm [(2\,\mathrm{sh}\,\sigma + \sigma\,\mathrm{ch}\,\sigma)\,\mathrm{sh}\,\vartheta\chi - \vartheta\chi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\chi] +$$

$$+ [\sigma\,\mathrm{ch}\,\sigma\,\mathrm{sh}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\varphi]\, U_\psi +$$

$$+ [(3\,\mathrm{sh}\,\sigma + \sigma\,\mathrm{ch}\,\sigma)\,\mathrm{ch}\,\vartheta\varphi - \vartheta\varphi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\varphi]\, V_\psi\,.$$

The positive sign holds again for the case when $\psi \geqq \varphi$, the negative one for the case when $\psi \leqq \varphi$.

$\bar{v}_{1m}$ is obtained from eq. (41) substituting $m\vartheta$ for ϑ and $m\sigma$ for σ. The expression for v_{0m} remains the same as for shear forces, i. e. (23).

3.4 Reaction in cross-beams in a grillage

The expression for reaction at the boundary $y = \pm b$ can be again written in the form

$$(42)\qquad \bar{Q}_{ym} = \sum_{m=1,2,3\ldots} p_{0m} \sin\frac{m\pi x}{l}\left[\kappa_{\alpha m} + \frac{m\pi b}{l}\,\frac{4\gamma_T}{\gamma_T+\gamma_P}\,\tau_{\alpha m}\right],$$

where $\kappa_{\alpha m}$ is given by formula (28), $\tau_{\alpha m}$ by formula (29) or formulae (23), (30) and (31) according to the interpolation formulae (32). For both of the latter cases the given expressions for reactions $\bar{Q}_{ym}$ represent only a check, since already the boundary conditions determine that for any position of the load the reaction at the free edge must be zero. Thus for example for $y = b$ and $-b \leqq e < b$ the coefficient $\bar{\nu}_{\alpha m}$ is equal to zero and for $e = b$ it is 1.

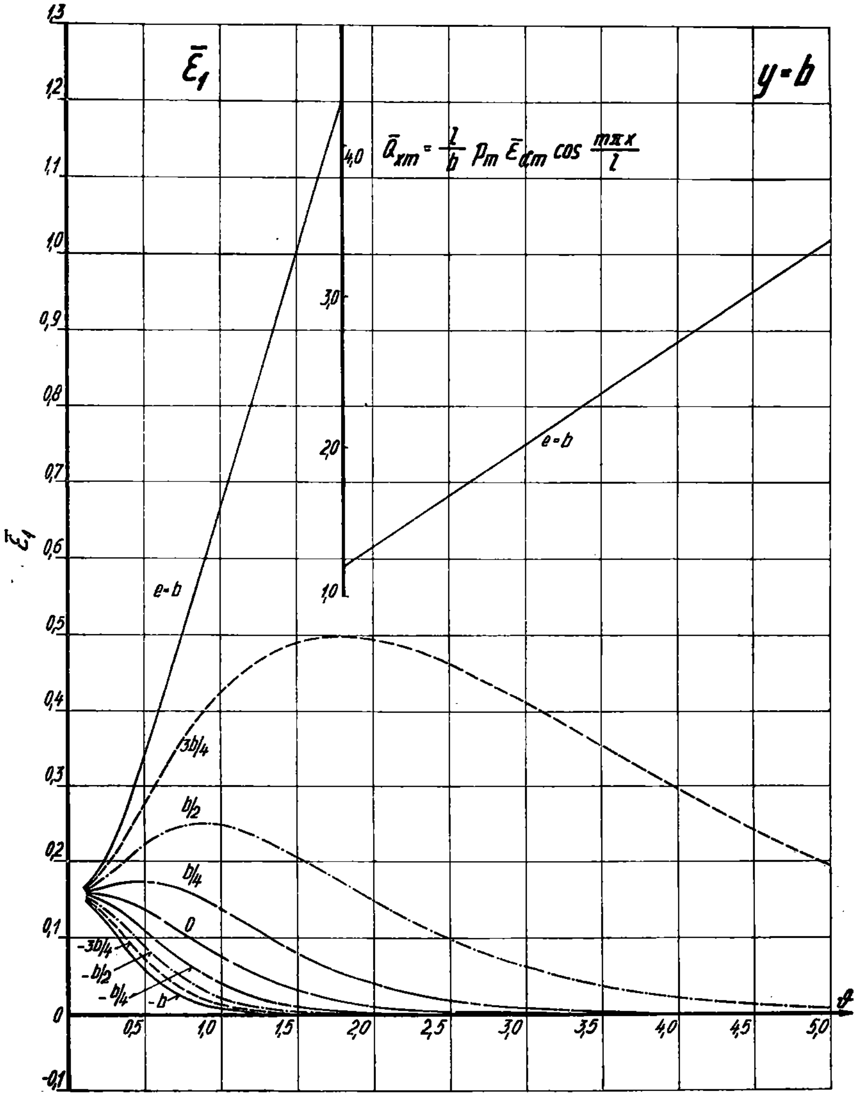

Fig. 28.

4. CONCLUSION

By deriving and calculation the values of coefficients for the determination of shear forces and reactions together with extended tables for finding deflections and bending moments both for $\eta = 0$ and $\eta = 0{,}15$ it is possible to use the Guyon-Massonnet method for a rapid, complete and accurate computation of bridge type grillages.

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APPENDIX 1

Diagrams expressing the variation of coefficients ε_0, ε_1 and $\bar{\varepsilon}_1$ for the calculation of shear forces and reactions in longitudinal direction ($\eta = 0$)

$$\varepsilon_{01} = \frac{\vartheta}{\sqrt{(2)}} \frac{1}{(\operatorname{sh}^2 2\lambda b - \sin^2 2\lambda b)} \{2 \operatorname{ch} \lambda(b \pm y) \cos \lambda(b \pm y)\, S_e + $$

$$+ [\operatorname{ch} \lambda(b \pm y) \sin \lambda(b \pm y) + \operatorname{sh} \lambda(b \pm y) \cos \lambda(b \pm y)\, T_e\} .$$

The upper signs hold for $e \geqq y$, the lower ones for $e \leqq y$.

$$\varepsilon_{11} = \frac{\vartheta}{4 \operatorname{sh}^2 \sigma} [2 \operatorname{sh} \sigma \operatorname{ch} \vartheta\chi + (2 \operatorname{sh} \sigma \operatorname{ch} \vartheta\varphi)\, U_\psi + (2 \operatorname{sh} \sigma \operatorname{sh} \vartheta\varphi)\, V_\psi] ,$$

$$\bar{\varepsilon}_{11} = \frac{\vartheta}{4 \operatorname{sh}^2 \sigma} \{(3 \operatorname{sh} \sigma - \sigma \operatorname{ch} \sigma) \operatorname{ch} \vartheta\chi + \vartheta\chi \operatorname{sh} \sigma \operatorname{sh} \vartheta\chi + [(5 \operatorname{sh} \sigma - \sigma \operatorname{ch} \sigma) \operatorname{ch} \vartheta\varphi +$$

$$+ \vartheta\varphi \operatorname{sh} \sigma \operatorname{sh} \vartheta\varphi]\, U_\psi + [(2 \operatorname{sh} \sigma - \sigma \operatorname{ch} \sigma) \operatorname{sh} \vartheta\varphi + \vartheta\varphi \operatorname{sh} \sigma \operatorname{ch} \vartheta\varphi]\, V_\psi\} ,$$

$$\bar{\varepsilon}_{01} = \varepsilon_{01} ,$$

where

$$S_e = [\mathrm{sh}\, 2\lambda b \cos \lambda(b \pm e)\,\mathrm{ch}\,\lambda(b \mp e) - \sin 2\lambda b\,\mathrm{ch}\,\lambda(b \pm e) \cos \lambda(b \mp e)]\,,$$

$$T_e = \{\mathrm{sh}\, 2\lambda b[\sin \lambda(b \pm e)\,\mathrm{ch}\,\lambda(b \mp e) - \cos \lambda(b \pm e)\,\mathrm{sh}\,\lambda(b \mp e)] +$$
$$+ \sin 2\lambda b[\mathrm{sh}\,\lambda(b \pm e) \cos \lambda(b \mp e) - \mathrm{ch}\,\lambda(b \pm e) \sin \lambda(b \mp e)]\}\,,$$

$$U_\psi = \frac{[(\sigma\,\mathrm{ch}\,\sigma - \mathrm{sh}\,\sigma)\,\mathrm{ch}\,\vartheta\psi - \vartheta\psi\,\mathrm{sh}\,\sigma\,\mathrm{sh}\,\vartheta\psi]}{3\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\sigma - \sigma}\,,$$

$$V_\psi = \frac{[(2\,\mathrm{sh}\,\sigma + \sigma\,\mathrm{ch}\,\sigma)\,\mathrm{sh}\,\vartheta\psi - \vartheta\psi\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\vartheta\psi]}{3\,\mathrm{sh}\,\sigma\,\mathrm{ch}\,\sigma + \sigma}\,,$$

$$\lambda = \frac{\pi\vartheta}{b\sqrt{2}}\,; \quad \sigma = \vartheta\pi\,; \quad \varphi = \frac{\pi y}{b}\,; \quad \psi = \frac{\pi e}{b}\,; \quad \vartheta = \frac{b}{l}\sqrt[4]{\left(\frac{\varrho_T}{\varrho_P}\right)}\,;$$

$$\alpha = \frac{\gamma_T + \gamma_P}{2\sqrt{(\varrho_T \varrho_P)}}\,; \quad \chi = \pi - |\varphi - \psi|\,.$$

For $|y| + |e| \leqq \frac{3}{4}b$ we have

$$\varepsilon_{\alpha m} = \varepsilon_{0m} + (\varepsilon_{1m} - \varepsilon_{0m})\,\alpha\,; \quad \bar{\varepsilon}_{\alpha m} = \bar{\varepsilon}_{0m} + (\bar{\varepsilon}_{1m} - \bar{\varepsilon}_{0m})\,\alpha$$

and for $|y| + |e| < \frac{3}{4}b$:

$$\varepsilon_{\alpha m} = \varepsilon_{\alpha m} + (\varepsilon_{1m} - \varepsilon_{0m})\sqrt{(\alpha)}\,; \quad \bar{\varepsilon}_{\alpha m} = \bar{\varepsilon}_{0m} + (\bar{\varepsilon}_{1m} - \bar{\varepsilon}_{0m})\sqrt{(\alpha)}\,.$$

Shear forces in longitudinal direction in a plate

$$Q_{xm} = p_{0m}\varepsilon_{\alpha m}\frac{l}{b}\cos\frac{m\pi x}{l}\,.$$

Shear forces in longitudinal direction in a grillage

$$Q_{xm} = p_{0m}\cos\frac{m\pi x}{l}\left(\frac{l}{2b\pi m}K_{\alpha m} + \frac{\gamma_P}{\varrho_P}\frac{m\pi b}{l}\mu_{\alpha m}\right).$$

Reaction in longitudinal direction in a slab

$$\bar{Q}_{xm} = p_{0m}\bar{\varepsilon}_{\alpha m}\frac{l}{b}\cos\frac{m\pi x}{l}\,.$$

Reaction in longitudinal direction in a grillage

$$\bar{Q}_{xm} = p_{0m}\cos\frac{m\pi x}{l}\left(\frac{l}{2b\pi m}K_{\alpha m} + \frac{2\gamma_P}{\varrho_P}\frac{m\pi b}{l}\mu_{\alpha m}\right).$$

Approximate value of shear force

$$Q_{xm} = K_{\alpha m} \frac{p_{0m}}{2bm} \frac{l}{\pi} \cos \frac{m\pi x}{l}.$$

The values of ε_{0m}, ε_{1m} and $\bar{\varepsilon}_{1m}$ are found from tables for $m\vartheta$.

APPENDIX 2

Diagrams expressing the variation of coefficients ν_0, ν_1 and κ_1 for the calculation of shear forces in cross direction ($\eta = 0$).

$$\nu_{01} = \pm \frac{1}{\mathrm{sh}^2\, 2\lambda b - \sin^2 2\lambda b} \{[\mathrm{ch}\, \lambda(b \pm y) \sin \lambda(b \pm y) +$$

$$+ \mathrm{sh}\, \lambda(b \pm y) \cos \lambda(b \pm y)]\, S_e + [\mathrm{sh}\, \lambda(b \pm y) \sin \lambda(b \pm y)]\, T_e\}.$$

The upper signs hold for $e \geqq y$, the lower ones for $e \leqq y$.

$$\nu_{11} = \frac{1}{4\, \mathrm{sh}^2\, \sigma} \{\mp 2\, \mathrm{sh}\, \sigma\, \mathrm{sh}\, \vartheta\chi + 2\, \mathrm{sh}\, \sigma\, \mathrm{sh}\, \vartheta\varphi\, U_\psi + 2\, \mathrm{sh}\, \sigma\, \mathrm{ch}\, \vartheta\varphi\, V_\psi\}$$

$$\kappa_{01} = \nu_{01}$$

$$\kappa_{11} = -\frac{1}{4\, \mathrm{sh}^2\, \sigma} \{\mp [(\sigma\, \mathrm{ch}\, \sigma - 2\, \mathrm{sh}\, \sigma)\, \mathrm{sh}\, \vartheta\chi - \vartheta\chi\, \mathrm{sh}\, \sigma\, \mathrm{ch}\, \vartheta\chi] +$$

$$+ [(\sigma\, \mathrm{ch}\, \sigma - 4\, \mathrm{sh}\, \sigma)\, \mathrm{sh}\, \vartheta\varphi - \vartheta\varphi\, \mathrm{sh}\, \sigma\, \mathrm{ch}\, \vartheta\varphi]\, U_\psi +$$

$$+ [(\sigma\, \mathrm{ch}\, \sigma - \mathrm{sh}\, \sigma)\, \mathrm{ch}\, \vartheta\varphi - \vartheta\varphi\, \mathrm{sh}\, \sigma\, \mathrm{sh}\, \vartheta\varphi]\, V_\psi\}.$$

The positive sign holds for $\psi \geqq \varphi$, the negative one for $\psi \leqq \varphi$. The expressions for S_e, T_e, U_ψ, V_ψ, λ, σ, φ, ψ, χ, ϑ and α are the same as in Appendix 1.

$$\nu_{\alpha m} = \nu_{0m} - (\nu_{1m} + \nu_{0m}) \sqrt{(\alpha)}\,; \quad \kappa_{\alpha m} = \kappa_{0m} + (\kappa_{1m} - \kappa_{0m}) \sqrt{(\alpha)}\,;$$

$$\tau_{\alpha m} = \tau_{1m} \sqrt{(\alpha)}\,.$$

Shear forces in cross direction in a slab

$$Q_{ym} = \nu_{\alpha m} p_{0m} \sin \frac{m\pi x}{l}.$$

Shear forces in longitudinal beams in a grillage

$$Q_{ym} = p_{0m} \sin \frac{m\pi x}{l} \left[\kappa_{\alpha m} + \frac{m\pi b}{l} \frac{2\gamma_T}{\gamma_T + \gamma_P} \tau_{\alpha m}\right].$$

Approximate value of shear force

$$Q_y = \kappa_{\alpha m} p_{0m} \sin \frac{m\pi x}{l}.$$

EINIGE ERGÄNZUNGEN ZUR METHODE DER BERECHNUNG VON TRÄGERROSTEN NACH GUYON-MASSONET

Eine der am besten geeigneten Methoden zur Berechnung von Rosten der Brückentype mit Anwendung der Analogie einer orthotropen Platte ist das Verfahren von Guyon-Massonnet [3, 6].

Für die Berechnung der Durchbiegungen, der Längs- und Querbiegungsmomente sowie der Torsionsmomente wurden von Massonet [6] Beziehungen für die Berechnung der Verteilungskoeffizienten abgeleitet. Der Autor der vorliegenden Arbeit hat Beziehungen sowohl für eine annähernde als auch für eine genaue Bestimmung der Schubkräfte und der Reaktionen in beiden Richtungen ermittelt, und zwar nicht nur in einer orthotropen Platte, sondern auch in einem Rost. In ähnlicher Weise wie Massonnet geht der Autor von zwei Grenzfällen je nach der Torsionsstarrheit der Konstruktion aus. Im ersten Fall, wenn die Torsionsstarrheit den Nullwert besitzt, wird eine geeignete Analogie angewendet, in der vorausgesetzt wird, dass der Querträger differentialer Breite als Träger auf elastischer Unterlage betrachtet werden kann; zu diesem Zweck wird das von Hetenyi [4] abgeleitete Verfahren angewendet. Im zweiten Fall, wenn die Konstruktion eine vollkommene Torsionsstarrheit aufweist, wird die Methode verwendet, die von Guyon für eine isotrope Platte entwickelt wurde [3].

Auf Grund der in der Arbeit abgeleiteten Formeln erfolgte die Berechnung des Wertes der Koeffizienten für die Ermittlung der Schubkräfte $\varepsilon_\alpha v_\alpha$ [2] für den Parameter der Querversteifung $\vartheta = 0{,}66874$ und den Parameter der Torsion $\alpha = 0{,}125$; $0{,}25$; $0{,}375$; $0{,}50$; $0{,}025$; $0{,}750$; $0{,}875$; $1{,}0$. Durch eine Analyse dieser Werte wurden die Interpolationsformeln 10 und 25 gefunden; diese ermöglichen die Bestimmung des Koeffizienten zur Berechnung der Schubkräfte für eine beliebige Konstruktion mit Hilfe der Grenzwerte der Koeffizienten für $\vartheta = 0$ und $\alpha = 1$ in Abhängigkeit vom tatsächlichen Parameter der Torsion α, von der Lage der Last und dem Querschnitt, in dem die Wirkung ermittelt werden soll. In den Diagrammen 2—28 sind die Zahlenwerte der Koeffizienten angeführt.

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